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QUALITY CONTROL AND RELIABILITY

TECHNICAL REPORT TR 6

SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY TESTING BASED ON THE WEIBULL DISTRIBUTION (RELIABLE LIFE CRITERION)

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OFFICE OF THE ASSISTANT SECRETARY OF DEFENSE

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Quality Control and Reliability

The content of this technical report was prepared on behalf of the Office of the Assistant Secretary of Defense (Installations and Logistics) by Professors Henry P. Goode and John H. K. Kao of Cornell University through the cooperation of the Office of Naval Research. It was developed to meet a growing need for the use of mathematically sound sampling plans for life and reliability testing where the Weibull Distribution adequately approximates the test data.

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FOREWORD

This technical report presents an acceptance-sampling procedure and tables of related sampling-inspection plans for the evaluation of lot quality in terms of reliable life or its complement, quantile life. Also included are tables of factors from which other sampling inspection plans of desired form can be determined and for use in evaluating the operating characteristics of specified plans. In addition, other associated tables of factors are included for determining the minimum lifetesting time required to provide a high level of assurance that reliable life requirements have been met. The Weibull distribution is used as a statistical model for item lifelength.

These procedures and tables have been prepared to supplement the plans and procedures previously issued in Department of Defense Technical Reports Number TR3[1] and Number TR4[2] for lot evaluation in terms of mean item life and in terms of hazard rate. These two plus this new report offer a comprehensive collection of tables for life and reliability testing based on the Weibull distribution.

The study upon which this report is based, as well as the work underlying the two previously issued, was done at Cornell University under a contract sponsored by the Office of Naval Research.

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SECTION I
INTRODUCTION

1.1 Summary

This technical report outlines an acceptance-sampling procedure and presents tables of related sampling-inspection plans for the evaluation of lot quality in terms of reliable life which is the life beyond which some specified proportion of the items will survive. Tables of plans are supplied for three reliability indices, .50, .90, and .99. The Weibull distribution, including the exponential and the Rayleigh distributions as special cases, is assumed as a statistical model for item lifelength. The evaluation of sample items is by attributes with lifetesting being truncated at the end of a specified period of time. Tables of conversion factors are also provided from which other sampling-inspection plans of desired form can be designed and for use in evaluating the operating characteristics of other specified sampling-inspection plans in terms of item reliable life.

A supplementary procedure and associated tables of factors are also included for use in determining the minimum lifetesting time required for sample items to provide assurance at a confidence level of .95 that the items in the lot or population meet the reliable life specified. Factors are provided for this alternative procedure for a representative range of sample sizes and acceptance numbers. Another table of factors are provided for lot evaluation under this procedure in terms of mean item life.

1.2 Introduction

The sampling inspection tables and procedures presented in this report evaluate item life for the lot in terms of reliable life which may be defined as the life beyond which some specified proportion of the items can be expected to survive. (A more precise definition will be found in Appendix A.) They have been prepared to supplement the Weibull plans and procedures for the evaluation of lot quality in terms of mean life and in terms of hazard rate at some specified life which were published as Department of Defense Quality Control and Reliability Technical Reports TR3[1] and TR4[2]. This and related material may also be found in reports by the authors published elsewhere.[3,4,5,6]

The papers previously published discuss the Weibull distribution at some length, review the underlying assumptions required, show the relationship between it and the exponential distribution, and offer much related material. Also, an extensive discussion of the Weibull distribution as a statistical model for lifelength, together with material on estimating the Weibull parameters can be found in a paper by Kao[7] and a paper by Plait[8]. Since this material is readily available, a general discussion of the Weibull distribution will not be repeated in this report.

It may be well to note, however, that the Weibull distribution has three parameters. One is a scale or characteristic life parameter commonly symbolized by the letter η. For the plans and procedures presented here this parameter is not of concern and need not be known or estimated; the methods are independent of its magnitude. Another is a shape parameter, conventionally symbolized by the letter β. This parameter is quite important for the tables and methods presented in this report; they depend directly on its magnitude. For appropriate application, the magnitude of β must be known or must be assumed to approximate some given value. Such knowledge is usually obtained either directly or indirectly from the analysis of past experimental and inspection results. The third parameter is a location or threshold parameter, commonly symbolized by the letter γ. For direct use of the procedures and tables presented here, it is assumed that this parameter has zero value; that there is no initial period of item life that is completely free of any risk of failure. For many applications this will be the case. However, if it is known that γ has some value other than zero, it is very easy to allow for this known value. This point will be discussed in a following section of the report and an illustrative example will be given.

Basic tables of conversion factors for the design of required acceptance plans or the evaluation of specified plans, and comprehensive tables of single-sampling acceptance inspection plans have been computed for an extensive range of β, or shape parameter, values. For the conversion factors, β values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 5 have been included. Tables of sampling inspection plans have been provided for the range of β values most commonly encountered with the specific values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, and $2\frac{1}{2}$ being included. It should be pointed out that β values of less than 1 apply to products whose hazard rate is relatively high in early life and which decreases with the passage of time. The smaller the value for β, the greater the rate of decrease. Such parameters seem to apply generally to a wide range of electronic components such as resistors and transistors. For a β value of exactly 1, the Weibull distribution is the same as the exponential; the exponential distribution is, in effect, a special case of the Weibull. At this parameter value the hazard rate is constant and independent of the passage of time. For β values greater than 1 the hazard rate is relatively low early in life and increases with the passage of time. For a β value of exactly 2, the Weibull distribution specializes to the Rayleigh distribution whose hazard rate increases linearly with time. The larger the value of β, the greater the rate of increase. This form of hazard rate pattern is typical of products for which failure is due to wear out or fatigue, as may be the case, for example, with ball bearings. Thus it should be obvious that the value for this parameter is critical and must be known for the appropriate application of a sampling inspection plan. This is true also, one should observe, in the case of the exponential distribution; i.e., ($\beta = 1$); it must be known or assumed the hazard rate is constant over time if exponential plans are to be appropriately applied.

For each β value included, factors and sampling plans have been computed for each of three reliability indices or proportions, namely .50, .90, and .99, selected for use in this study to define reliable life. It can readily be seen that the reliable life ρ_r is equivalent to the quantile of order $(1-r)$ of a distribution (see Reference 9, p. 181). If the reliability index r is the specified survival probability at time ρ, then the reliable life ρ_r is the theoretical lifelength associated with r. For example, for a product if $\rho = 1000$ hours and $r = .90$, 90% of the items can be expected to have a life of 1000 hours or longer. Hence if r is chosen to be close to unity, ρ_r will be close to zero. On the other hand, if r can be tolerably small, then ρ_r can be very large indeed. The two trivial cases have been omitted here for $r = 1$ and 0 for which $\rho_r = 0$ and ∞ respectively. A special case of ρ_r is found when $r = \frac{1}{2}$, then the measure ρ_r is known as median life, ξ.

A notable area of application of the reliable life concept can be found in the anti-friction bearing industry in which the rated bearing life for a given application is usually the reliable life with the reliability index r set equal to 90%. A bearing manufacturing firm, for example, lists their bearing capacities based upon LB-10 Life ($\rho_{.90}$) equal to 3000 hours and a speed of 500 rpm. If a life of other than 3000 hours is desired or the actual speed is different from 500 rpm, the load capacity can be appropriately adjusted by using one of the so-called trade-off or acceleration factors similar to those well-known in the electronic component industry.

(Unfortunately, this information for major electronic components is still not widely available.)

Another example of application employing the concept of reliable life is found in the area of biological assaying where, for example, the efficacy or potency of a poisonous material such as an insecticide or a herbicide is characterized by its median lethal dose, LD50[10] which is the theoretical dosage corresponding to the insect's or plant's reliable life with r = 50%, commonly known as its median life.

Numerous other areas of application can be cited; e.g., fatigue testing of metals or components, sensitivity testing of fuzes or primers, and breakdown voltage testing of dielectric materials or insulators, to name just a few. For this reason, the examples in this report which demonstrate the use of various tables will not be restricted to any specific area of application, although the report is directed mainly to the areas of fatigue testing and biological assaying.

In the area of fatigue testing, including testing to failure of most anti-friction bearings, where the fatigue life for p % survival[11,12] is exactly equal to the reliable life for r = p/100, the Weibull distribution is found to be particularly useful[13,14]. On the other hand, in the area of biological assaying, although the lognormal distribution has been traditionally used[10], the Weibull distribution which can be made to have a shape similar to a lognormal distribution should prove to be equally useful.

1.3 The Form of the Acceptance Procedure

The following acceptance sampling-inspection procedure has been assumed for the plans and methods presented in this report:

(a) Select at random a sample of n items from the lot.

(b) Place these sample items on life test for some preassigned test time t.

(c) Determine the number of items that fail prior to the termination of the test (at time t).

(d) Compare the number of items that fail with an acceptance number c specified for the selected plan. If the number that fail is equal to or less than the acceptance number, accept the lot; if the number that fail exceeds it, reject the lot.

Lifelength measurements and specifications, and the test period, t, may be in any appropriate measurable units -- minutes, hours, or stress cycles endured, for example. While only single-sample acceptance plans are included in this report, double-sampling or multiple-sampling plans may be constructed, if desired, through use of the basic conversion ratios provided.

SECTION 2
THE BASIC CONVERSION FACTORS

One may note that the acceptance procedure is of the familiar attribute form. The only modification is that the item quality of interest is life and that testing for life is truncated at some time t. Thus the lot is effectively evaluated in terms of the proportion of items, p', that can be expected to fail before the test truncation time. With the shape parameter, β, of the distribution known or given and with the test time, t, specified, this proportion, p', is a function only of the reliable life for the lot, ρ, and the reliability index, r, of the lot that is to have this reliable life. Hence the operating characteristics of any specified sampling-inspection plan depend only on t, ρ, and r (for any given value for β). So that the ratios and sampling plans will be convenient for general use, the dimensionless quantity t/ρ has been employed; one does not have to work in terms of specific values for test truncation time and reliable life. In practical application it will be found to be quite easy to convert from a ratio to specific values of t and ρ, or from specified values for these measures to the equivalent ratio. However the proportion r that must have the specified reliable life, ρ, could not be treated in this convenient manner. It has been necessary to compute separately basic factors and tables of plans for each of the selected values of r. As previously mentioned, these are r = .50, r = .90, and r = .99.

As a foundation for the reliable life plans included in this report, tables of basic conversion factors have accordingly been computed to show for the Weibull distribution the relationship between p' and the ratio t/ρ. These factors may also provide a basis for the design of other sampling inspection plans for reliable life using techniques commonly employed with the binomial, hypergeometric, or Poisson distributions in the design of ordinary attribute plans. Also, the conversion factors may be used to evaluate plans in use or ones that have been specified for use. Examples of such applications will be given.

These tables of factors will be found at the end of this report as Tables 1-a,-b,-c, and 2-a,-b,-c, with $r = .50$ for a, $r = .90$ for b, and $r = .99$ for c. For convenience in tabulation and use, the value $(t/\rho) \times 100$ has been employed rather than t/ρ, and p' is expressed as a percentage rather than as a decimal fraction. Tables 1 list values for $(t/\rho) \times 100$ for specified values of $p'(\%)$. Tables 2 list values for $p'(\%)$ for specified values of $(t/\rho) \times 100$. In each case, separate tables have been prepared for each of the selected values for r. These two sets of tables are meant to supplement each other so as to provide convenient conversion in either direction. Note also that by the provision of these two supplementary sets, a considerably wider range of conversion values is provided; the factors in one table are expanded in range in the region where they are compressed in the other table, and vice versa. The values selected for $p'(\%)$ and $(t/\rho) \times 100$ from which to convert have been determined by the use of a standard preferred number series. Details of the mathematical steps involved in establishing the $(t/\rho) \times 100$ and p' relationships will be found in the appendix.

Example (1)

A sampling inspection plan is required for the evaluation of production lots of a product in terms of reliable life, with reliable life defined as the life beyond which 50% of the items can be expected to survive. A reliable life of 1000 hours is considered acceptable and for lots with this reliable life or longer the probability of acceptance should be high, say .95 or more. A reliable life of 400 hours is considered unacceptable so that lots with this reliable life or less should have a low probability of acceptance, say .05 or less. A test truncation time of 100 hours is to be employed. Experience has indicated the Weibull distribution applies with a value for the shape parameter of $1\frac{2}{3}$ and for the location parameter o 0. Thus, $\rho = 1000$ at the AQL (acceptable quality level) for which $P(A) \geq .95$, $\rho = 400$ at the RQL (rejectable quality level) for which $P(A) \leq .05$, $r = .50$, $t = 100$, $\beta = 1\frac{2}{3}$, and $\gamma = 0$.

Through the use of Table 2a which contains conversion factors for $r = .50$, values for p' at the AQL and the RQL can be determined. For the values for t and ρ specified,

$$(t/\rho) \times 100 = (100/1000) \times 100 = 10 \qquad \text{(at the AQL)}$$

$$(t/\rho) \times 100 = (100/400) \times 100 = 25 \qquad \text{(at the RQL)}.$$

By entering Table 2a with these two values and reading from the column for the shape parameter value, β, of $1\frac{2}{3}$, it is found that at the AQL $p' = 1.48(\%)$ and at the RQL $p' = 6.45(\%)$. These are the respective probabilities of item failure before the end of the 100 hour testing period.

With these two values for p', values for n, the sample size, and c, the acceptance number can be determined through any of the well-known methods ordinarily used in the design of attribute sampling inspection plans. The Poisson-based tables prepared by Cameron[15] will serve well for this example. Through use of these tables it is found that an acceptance number, c, of either 4 or 5 will meet the requirements for p' reasonably well. Through further use of Cameron's tables and with an acceptance number of 5, it is found that a sample size of 164 will provide the required consumer's risk. With this acceptance number and sample size, the tables indicate the probability of acceptance at the acceptable quality level will be between .95 and .975 so that the producer's risk requirement will also be met. An alternative procedure for determining c and n and one that is somewhat more precise is to use a beta probability chart (which is based on the binomial distribution). One may be found in a paper by Kao.[16]

Example (2)

For another application of sampling inspection in terms of reliable life, a Military Standard Plan has been specified, one with an AQL of 1.5% and with Sample Size Code Letter K. For single sampling, the sample size for this plan is 110 items and the acceptance number is 4. Reliable life has been defined in this case as the life beyond which 90% of the items can be expected to survive; i.e., $r = .90$. The testing of sample items is to be truncated at 400 hours. The Weibull distribution can be assumed as a lifelength model with $\beta = 2\frac{1}{2}$ and $\gamma = 0$. The user of this plan would like to know what its operating characteristics are in terms of reliable life and in particular what protection he as the consumer will receive.

To determine these characteristics, the first step is to determine for the values for n and c specified the corresponding p' values associated with appropriate probabilities of acceptance. These values for p' may be obtained approximately by reading them from the Operating Characteristic curves supplied as a part of the MIL-STD-105C Plans or by use of cumulative tables of the Poisson or binomial distributions. Examination of the operating characteristic curve for the selected plan supplied in the 105C Standard indicates that at $P(A) = .95$, $p' = 1.8\%$ and at $P(A) = .10$, $p' = 7.3\%$ (approximately, in both cases). A check by means of Poisson tables will indicate these values are reasonably close to the right percentages.

The next step is to use these percentages to determine from Table Ib, which gives tables of conversion factors for $r = .90$, the corresponding $(t/\rho) \times 100$ values. With these values and with the value for t specified, only a simple computation is required as the final step necessary to find the desired reliable life values. At the acceptable quality level for which $p' = 1.8(\%)$, through interpolation in the column of factors for $\beta = 2\frac{1}{2}$, a value for $(t/\rho) \times 100$ of 49.4 can be found. With $t = 400$, $(400/\rho) \times 100 = 49.4$ or $\rho = 810$ hours. This, then, is the "acceptable" reliable life; the reliable life required for the lot if the probability of acceptance is to be high. At the unacceptable quality level for which $p' = 7.3(\%)$, interpolation in Table Ib will give a value of 87.5 for $(t/\rho) \times 100$. Substitution of $t = 400$ gives $(400/\rho) \times 100 = 87.5$ or $\rho = 460$ hours. Thus if the reliable life for a lot is 460 hours or less the probability of acceptance will be low, namely .10 or less.

Under the use of MIL-STD-105C plans as selected for this example, the alternatives of double-sampling and multiple sampling are available. If double sampling is employed, for example, for Sample Size Letter K the first sample size would be 75 and the second 150. For an AQL of 1.5% the acceptance number would be 2 for the first sample and the rejection number 8. For failures from the combined samples the acceptance number would be 7 and the rejection number 8. All other elements of the procedure for double-sampling would be employed. The test time for the first sample would be 400 hours, the same as for single sampling; likewise the test time for the second sample would have to be 400 hours. One may note that a possible reduction under double-sampling in the number of sample items that may have to be inspected can be achieved only by a doubling of the duration of the life-testing time for some lots. Under double sampling or multiple sampling employing the same Sample Size Code Letter and AQL, the operating characteristics will obviously be closely the same as for single sampling.

Example (3)

Suppose that in another application the requirements for lot quality and the inspection conditions are the same as for Example (2) with the exception that γ, the location or threshold parameter, is equal to 250 hours instead of 0. As before, at $P(A) = .95$, $p' = 1.8\%$ and the corresponding $(t/\rho) \times 100$ value is 49.4. Likewise, the $(t/\rho) \times 100$ value at $P(A) = .10$ for which $p' = 7.3\%$ is 87.5. However, now time values must be considered in terms of $\gamma = 250$. A new value t_0, which is $t_0 = t - \gamma = 400-250 = 150$ must be computed and used in working with the factors from the table. At the acceptable quality level now $(t_0/\rho_0) \times 100 = 49.4$ or $(150/\rho_0) \times 100 = 49.4$ which results in a value for ρ_0 of 300 hours for the relative reliable life. This may be converted back to absolute or real terms by simply adding the value for γ; thus $\rho = 300 + 250 = 550$ hours for the acceptable reliable life. At the unacceptable quality level, $(t_0/\rho_0) \times 100 = 87.5$ or $(150/\rho_0) \times 100 = 87.5$ which results in a value for ρ_0 of 170 hours. The real or absolute value for the unacceptable reliable life is $\rho_0 + \gamma$ which is $170 + 250$ or 420 hours. In any application of the sampling plans or basic conversion factors presented in this report, when γ has some value greater than 0, all that must be done is to work with the tabulated values in terms of t_0 and ρ_0 where $t_0 = t - \gamma$ and $\rho_0 = \rho - \gamma$. The solution in terms of t_0 or ρ_0 may then be converted back to absolute or real terms by adding the value for γ.

SECTION 3
THE TABLES OF SAMPLING PLANS

3.1 Description of the Tables of Plans

This report also includes twenty-four tables of sampling inspection plans. These tables cover eight values of the shape parameter, β, over the range most frequently encountered in practice. For each β value, tables have been prepared for each of the three values of the reliability index, r, for which the relationship between ρ' and $(t/\rho) \times 100$ has been determined. These tables, Tables 3a1 through 3c8, will be found at the end of the report.

Each table lists for a range of acceptance numbers, c, the minimum sample size, n, to be employed. A plan, or pair of c and n values, is available for a variety of $(t/\rho) \times 100$ ratios and for each ratio, for acceptance numbers ranging from 0 to 10. The plans have been designed so that if 100 times the ratio between the test-truncation time, t, and the reliable life for the lot, ρ, is equal to the ratio value in the selected column heading, the probability of acceptance, P(A) will be .10 or less. That is, a selected plan assures with 90% confidence or more the rejection of lots for which the $(t/\rho) \times 100$ ratio is equal to or greater than the value shown in the column heading. It has been assumed that in the use of these plans the consumer's risk will be of most importance. For this reason the plans have been cataloged by their $P(A) \leq .10$ ratios. These ratios (as shown in the column headings) are a common measure of consumer protection and may be regarded in the same way as LTPD (lot tolerance per cent defective) values are regarded in describing the operating characteristics of ordinary attribute or variables acceptance plans.

In addition, for each of the plans the $(t/\rho) \times 100$ ratio has been determined for which the probability of acceptance is .95 or more. Each of these

$P(A) \geq .95$ ratio values will be found enclosed in parentheses immediately under the corresponding sample size number. These ratio values may be regarded in the same way that AQL (acceptable quality level) values are as a measure of the producer's risk. If the item life distribution for a lot is such that its $(t/\rho) \times 100$ ratio is equal to or less than the table heading value, the selected plan assures a $P(A) \geq .95$.

Thus the two ratio values, the one in the column heading and the one in parentheses immediately below the sample size number, describe in broad terms the operating characteristics of each plan. If one or the other of these values is specified for an acceptance inspection application, with additional information, a suitable plan may be selected from the tables. Alternatively, the pair of values may be used to determine in approximate terms the operating characteristics of a plan that has been specified or that is in use and whose values for n and c match reasonably well one of the plans in the tables.

To make these plans available for general use, the binomial distribution and the Poisson distribution were employed in their design. Binomial tables prepared by Grubbs[17] were used in the design of all plans using acceptance numbers, c, up to 9 and sample sizes, n, up to 150. The remainder of the plans, those for c = 10 and for sample sizes over 150, were designed by employing the Poisson distribution as an approximation to the binomial. Here use was made of np' values prepared by Cameron.[14] In each case of changing from the binomial to the Poisson distribution, the match in sample sizes was checked. It was found to be close in all cases. Furthermore, the slight differences that were found were on the conservative side; the sample size under the Poisson was slightly larger than the number theoretically required under the binomial assumption.

3.2 Use of the Plans

In making use of the plans, one should recognize that the binomial and Poisson distributions were employed in their design. For this reason the size of the sample should be relatively small compared to the size of the lot, just as in the case of other published tables of attribute sampling-inspection plans. If the sample size is relatively large, the probability values assigned to the $(t/\rho) \times 100$ ratios will not precisely apply. This point, however, should present little difficulty in practice.

In addition to making sure the sample size is not so large that it constitutes a substantial portion of the lot, a few other practical points in application should be observed. One is that if specified sample sizes are for practical reasons to be rounded off to the nearest number ending in five or zero (or to the nearest one hundred), this rounding off should be to a number larger than the number given in the table. This will assure the retention of the specified consumer's protection, $P(A) = .10$ or less. Another point of practice that should usually be followed is that if a plan is not available for which the $(t/\rho) \times 100$ ratio in the column heading matches closely the desired ratio, a plan should be selected from the column with the next smaller ratio value. By following this conservative practice a confidence level of 90% or greater will be maintained in assuring that the specific minimum reliable life has been met. On the other hand, if some acceptable quality level must be guaranteed (a ratio or a reliable life for which $P(A) \geq .95$) and a matching ratio value is not available in the body of the tables, a plan with the next higher value should be used. If this is done, a lot with an acceptable reliable life will have $P(A) \geq .95$. One should also note that when plans with the desired ratios are not available in the tables, interpolation may be employed between the listed sample sizes to find a new plan that does have more nearly the desired operating characteristics. Finally, it should be noted that testing of sample items for lots that are to be rejected can be terminated after the acceptable number of failures has been exceeded. The lot is to be rejected and so further testing will be of little use unless the sampling inspection data is to be used to provide an estimate of the process average for the product or the vendor. In the latter case, testing should continue for the full period, t.

Example (4)

A sampling inspection plan for a product is required which will accept with a probability of .10 or less lots whose reliable life is 400 hours or less. In this application reliable life has been defined as the life beyond which 90% of the items in the lot will survive ($r = .90$). The user would also like to be able to assure the producer of the product that if the reliable life for a lot is 2,000 hours or more, the probability of acceptance will be high, say .95 or greater. A test period of 200 hours is to be employed. Through past experience with the product it has been established that the one-parameter exponential distribution applies for item lifelength, i.e., the Weibull distribution with the value for β, the shape parameter, being 1 and for γ, the threshold parameter being 0.

With these specifications for the sampling plan, 100 times the ratio of the test time, t, to the reliable life, ρ, is $(200/400) \times 100$ or 50 at the unacceptable reliable life of 400 hours for which $P(A) \leq .10$ has been specified. At the acceptable reliable life of 2,000 hours the $(t/\rho) \times 100$ ratio is $(200/2,000) \times 100$ or 20. An inspection plan meeting these ratio requirements will be found in Table 3b4 which lists plans for $\beta = 1$ and $r = .90$. Any plan in the fifth column (headed 50) will meet the unacceptable reliable life specification. Of the plans assigned to this column, the last one has a ratio value (in parentheses) of 20, the value required at the acceptable reliable life. The plan is thus to use a sample size, n, of 301 and an acceptance number, c, of 10.

Example (5)

A plan has been specified for the acceptance inspection of a product which requires that a sample of 375 items be drawn from the lot and tested for 500 hours. If no more than 7 items fail before the end of the test period, the lot is to be accepted; if more than this number fail, it is to be rejected. Data from past inspection indicates a value for the shape parameter, β, of $\frac{2}{3}$ applies with the location parameter, γ, being 0. The user of this plan would like to know what its operating characteristics are in terms of reliable life, with reliable life being defined as the median life or the life beyond which 50% of the items can be expected to survive.

An answer may be found by inspection of Table 3a3 which tabulates plans for $\beta = \frac{2}{3}$ and $r = .50$. An examination of this table indicates a plan is tabulated approximating the one to be used, the plan for $c = 7$ and $n = 372$. For this plan the $(t/\rho) \times 100$ value for which $P(A) \leq .10$ is found (in the corresponding column heading) to be 1.0. By the substitution of the test period specified, 500 hours, for t in this ratio, one obtains $(500/\rho) \times 100 = 1.0$ or $\rho = 50{,}000$ hours. Thus if the reliable life is 50,000 hours or less, the probability of acceptance will be .10 or less. For this plan the ratio value at the acceptable reliable life is .19 (as shown by the number in parentheses under the sample size 372). By substitution of the specified value for t into the ratio, $(500/\rho) \times 100 = .19$ or $\rho = 263{,}000$ hours is obtained. This is the reliable life for which the probability of acceptance will be .95.. These two values for reliable life describe in a practical way the operating characteristics of the plan that has been specified.

Example (6)

For a sixth example consider a case for which an underlying Rayleigh distribution can be assumed, i.e., a special Weibull distribution with the shape parameter, β, equal to 2 and for which the threshold parameter, γ, equal to 1200 cycles. A plan is required for which the P(A) = .10 or less if the reliable life is 6,000 cycles or less with reliable life being defined as the life beyond which 99% of the items will survive. A test truncation time of 5,000 cycles seems reasonable and could be used. The user would also like to know the effect of cutting the test time to 3,000 cycles.

In the selection of a plan reference must be made to Table 3c7 which tabulates plans for β = 2 and for r = .99. The first step is to convert the specified values for t and ρ to relative values in terms of $\gamma = 0$. Thus $t_0 = 5{,}000 - 1{,}200 = 3{,}800$ cycles and $\rho_0 = 6{,}000 - 1{,}200 = 4{,}800$ cycles. The $(t_0/\rho_0) \times 100$ ratio is $(3{,}800/4{,}800) \times 100 = 79$ or approximately 80. Any plan in the column with this ratio heading in the table of plans will meet the rejectable quality level requirements. One possibility is the plan for which n = 349 and c = 0. This provides the minimum sample size that can be used.

The proposal to cut the test time to 3,000 cycles may now be considered. In this case $t_0 = 3{,}000 - 1{,}200 = 1{,}800$ cycles and $\rho_0 = 6{,}000 - 1{,}200 = 4{,}800$ cycles. The $(t_0/\rho_0) \times 100$ ratio is now $(1{,}800/4{,}800) \times 100 = 38$. The nearest ratio available in the table is 40. In the column with this ratio heading, the best plan available (from the standpoint of sample size) is the one for which c = 0 and n = 1400. The penalty for reducing the test period is thus to increase the sample size from 349 to 1400.

3.3 Choice of Acceptance Number

It will be instructive to compare the two possibilities discussed in the above example in terms of the acceptable reliable life, the life for which P(A) = .95. For the first one for which c = 0, n = 349, and $t = 5{,}000$, the ratio at the AQL is 12 (as shown by the figure in parentheses in the body of the table of plans). Thus $(t_0/\rho_0) \times 100 = 12$ or $(3{,}800/\rho_0) \times 100 = 12$ from which one determines that $\rho_0 = 32{,}000$ cycles. Converted back to absolute terms, $\rho = \rho_0 + \gamma = 32{,}000 + 1{,}200$ or 33,200 cycles. This must be the reliable life if the lot is to have a high probability of acceptance. For the second possibility for which n = 1400, c = 0, and t = 3,000, the ratio at the AQL is 5.9. Thus $(1{,}800/\rho_0) \times 100 = 5.9$ or $\rho_0 = 31{,}000$ cycles. Converted to absolute terms, $\rho = 31{,}000 + 1{,}200$ or 32,200 cycles which is approximately the same AQL requirement as for the first plan. Thus it should make no difference to the producer which plan is used. These computations just made illustrate a unique feature of the Weibull plans for life and reliability testing; the ability of a plan to discriminate between good and bad lots depends on the size of the acceptance number rather than on the size of the sample (as is the case for ordinary attribute sampling plans). For any given acceptance number (given some value for β) a nearly constant ratio will be found between the acceptable reliable life and the unacceptable reliable life regardless of the general level of these lives and regardless of the sample sizes specified. This will also be the case regardless of the value chosen for the proportion r that must survive.

This point has been more fully discussed in the author's reports for the Weibull mean life plans[1,3] and the Weibull hazard rate plans.[2,4] Table 4 of Reference 1 (or alternatively Table 3 of Reference 3) gives approximate values for $\mu_{.95}/\mu_{.10}$. These same ratios can be used for the reliable life plans presented here, that is, they can be used as $\rho_{.95}/\rho_{.10}$ values. If both the acceptable reliable life with P(A) = .95 and the unacceptable reliable life with P(A) = .10 are specified, use of this table of values will indicate at once what the acceptance number should be. This information provides a very helpful start in designing a plan to meet given needs. In the above application, for example, if an acceptable reliable life of 33,000 cycles had been specified, $\rho_{0(.95)}/\rho_{0(.10)}$ = 31,800/4,800 or 6.7. Reference to the table just described would indicate that for $\beta = 2$, the acceptance number c would have to be 0. On the other hand, if an acceptable reliable life of 12,000 cycles had been specified instead (for which ρ_0 = 12,000 - 1,200 or 10,800), the $\rho_{0(.95)}/\rho_{0(.10)}$ ratio would be 10,800/4,800 or 2.2. Reference to the table would indicate the acceptance number must be 3. Reference again to Table 3c7 of this report would indicate the sample size must accordingly be 1,010 if the test period is to be 6,000 cycles (in which case the t/ρ ratio is 80) or must be 4,050 if the test period is to be 3,000 cycles (in which case the t/ρ ratio is 40).

SECTION 4

LIFE TESTING TIME REQUIREMENTS TO ASSURE REQUIRED RELIABLE LIFE

4.1 The Tables of Lifetesting Times

For many reliability or lifelength evaluation applications it may be most useful to employ an acceptance procedure of a form different than that outlined in the preceding section. This is to simply determine the minimum test time required (for some specified or selected small acceptance number -- which may be zero) to provide a high degree of confidence that the items in the lot or population meet the specified lifelength requirements. Such a procedure would seem to be particularly helpful in the many cases currently encountered for which item lifelength is relatively long and at the same time for which the sample lifetesting time must be relatively short for lot evaluation to be economically and chronologically feasible. For this reason the following tables of factors for easily determining the minimum lifetesting times required have been compiled. Factors have been determined for each acceptance number from zero through five.

Tables 4-a,-b,-c list in terms of multiples of the specified reliable life the minimum lifetesting times required to assure for accepted lots lot compliance with specifications. Reliable life is again defined as the life beyond which some specified proportion of the items in the lot can be expected to survive. Table 4-a provides values for cases for which the proportion expected to survive is .50. Table 4-b provides values for cases for which the desired proportion surviving or the reliability index is .90; Table 4-c provides values for which the proportion or reliability index required is .99.

In Table 5 are tabulated lifetesting times for use to make an evaluation in terms of mean item life. The values given are multiples of the required or specified mean item life that must be employed as sample lifetesting times to assure lot or population compliance. Mean life is another alternative life-quality measure that may be useful in a wide variety of applications.

Within each of these tables, values will be found for seven different values for the shape parameter: $\beta = \frac{1}{2}, \frac{3}{4}, 1$ (the exponential case), $1\frac{1}{3}$, $1\frac{2}{3}$, 2 (the Rayleigh case) and $2\frac{1}{2}$. For each β value, testing time values have been tabulated for acceptance numbers, c, of 0, 1, 2, 3, 4, and 5. For each possible pair of c and β values, testing times are listed for sample sizes, n, of 10, 25, 50, 100, 250, 500, and 1000 items. It is expected that these ranges of values for r, β, c, and n will encompass those values most commonly required in reliability and lifetesting practice. For cases for which values specifically required for n or β are not listed but are within the range covered by the tables, interpolation may be employed to find the required minimum testing time provided one understands that only an approximation to the specified level of confidence (.95) will be obtained.

4.2 Use of the Tables

The values tabulated in the body of Table 4-a,-b,-c are multiples of the specified reliable life (or specified mean life for Table 5) that must be used as a testing time for sample items to assure lot compliance with a confidence level of .95. If no more than the specified number of items, c, fail before the end of this testing time, it may be inferred, with this confidence level, that the reliable life (or, alternatively, the mean item life) for the lot is equal to or greater than the required or specified value. The meaning and use of these tables of values can be better described through the several simple examples that follow.

Example (7)

A sampling inspection plan for acceptance is required for a certain electronic component purchased from time to time in some quantity. Past experimental and inspection data indicates the Weibull distribution applies as a lifelength model and that a value for the shape parameter, β, of approximately $\frac{3}{4}$ and for the location parameter, γ, of 0 can be assumed. Each lot is to be evaluated in terms of a required reliable life of 2,000 hours, with reliable life defined as the life beyond which 90% of the items in the lot will live. That is, r is equal to .90. Testing facilities are available for testing 100 items at a time. The lot size and the costs of inspection per item are such that a sample of this size can be economically justified. A decision on each lot should be reached as quickly as possible and for this reason the duration of the lifetesting time must be kept as short as possible.

The necessary minimum lifetesting time is found by reference to Table 4-b which lists values for r, the proportion that must survive beyond the reliable life, of .90. Under the section of this table for $\beta = \frac{3}{4}$, it is found that for an acceptance number, c, of 0, the time must be .18 times the required reliable life. Since this has been specified as 2,000 hours, the testing time must be .18 x 2,000 or 360 hours. The acceptance number, c, of 0 is used since the duration of the lifetesting time must be kept short; use of larger acceptance numbers will require longer testing times.

Thus if 100 items are drawn at random from a lot and put under life test, and if no items fail before the end of 360 hours, the lot may be accepted at that time as meeting the reliable life specification. One may be 95% confident that 90% or more of the items in the lot will have a life of at least 2,000 hours. A slightly different way of expressing this is that one may be 95% confident that the life beyond which 90% of the items will survive is 2,000 hours or more.

Example (8)

A lifelength evaluation is to be made for a new source of supply for a product. A value of $1\frac{1}{3}$ can be assumed for β, the shape parameter, and a value of 0 for γ, the location parameter. For the supply of product to be suitable for use, the mean item life must be at least 400 hours. Because of the unit cost of the item, the sample size should be kept small -- preferably not over 25 items. However, relatively long test times can be tolerated. For this reason an acceptance number of 5 together with the comparatively long test time that will be required will be used. It is expected that the more extensive test experience that will be accumulated will provide a better overall evaluation of the product.

Examination of Table 5 which lists values for use in mean life evaluation indicates for $\beta = 1\frac{1}{3}$, $n = 25$, and $c = 5$ that the lifetesting time must be .62 times the required minimum mean life. Hence for the source of supply to be acceptable, no more than 5 items must fail before the end of $.62 \times 400$ or 248 hours.

Example (9)

For cases for which the location or threshold parameter, γ, -- the lifetime below which there is no risk of item failure -- is greater than zero, the following procedure may be used: (a) subtract the value for γ from the required reliable or mean life to get a converted value in terms of $\gamma = 0$, (b) multiply this converted value by the factor selected from the table (in the usual way) to get a lifetesting time in converted terms, and (c) add the value for γ to this testing time to get the required testing time in absolute terms. The following example will illustrate this simple variation in technique.

Consider an application for which the Rayleigh distribution or Weibull distribution with a value of 2 for β, the shape parameter, may be assumed.

Past experience with the item in question indicates a value for the threshold parameter, γ, of 3,000 cycles should be expected. The sample size, n, is to be limited to 50 items; the acceptance number, c, to 1 item. The minimum reliable life that can be tolerated is 5,000 cycles, with reliability index, r, equal to .99. The minimum lifetesting time for sample items to assure lot compliance with 95% confidence is required.

Subtraction of the value for the location parameter from the required reliable life gives 5,000 - 3,000 or 2,000 cycles as a converted value for required reliable life. From Table 4-c in which time values for $r = .99$ are tabulated, a value of 3.0 is found for $\beta = 2$, $n = 50$, and $c = 1$. The required testing time in converted terms is thus $3.0 \times 2,000$ or 6,000 cycles. Addition of the value for γ, 3,000 cycles, to this converted value gives 6,000 + 3,000 or 9,000 as the minimum number of cycles required in absolute or real terms.

4.3 Choice of Sample Size and Acceptance Number

The size of sample that will be most suitable for an application will depend on a number of factors. One is the unit value of an item. If this is high and the usefulness of the item is impaired or destroyed by testing, the sample size will have to be relatively small. A related factor is the size of the lot. If it is small and items are made useless by testing, the sample size must again be kept smallfor practical and economic reasons. Another factor in making a choice is the amount of lifetesting facilities available. The sample size may have to be limited to the number of testing positions available. A fourth and important factor is the period of time available for conducting the life tests. If a decision must be reached quickly because the items are urgently needed, the required test time may be minimized by employing a relatively large sample size. For many components currently in use, the required reliable or mean life is many hundreds of thousands of hours. For these products sample sizes must of necessity be quite large; if not, the testing period required may be many months or years. Another and related factor is the unit-hour cost of lifetesting; this may be high because costly test facilities are required. In such cases the total unit-hours of testing must be minimized by suitable choice of sample size and test duration. In addition to the factors just listed, many other minor related factors may have to be considered such as the current availability of existing test facilities.

No systematic method for determining the most economical or most satisfactory sample size is available at this time. However, a somewhat reasonable decision may be made in most applications of the procedures described in this report by the use of some judgment together with some rough inspection cost estimates for various alternatives. One may first examine the test time values tabulated in the tables for each of the sample sizes listed. Then by consideration of the costs -- both those reducible to money terms and those not reducible -- associated with each item in the sample and the costs associated with each unit of required test time, one may determine, for example, that a sample as small as 10 or 25 items is clearly too small and that one of 500 or 1,000 is clearly too large and that one of 100 items is perhaps reasonably close to the "optimum" size.

It should be noted at this point that nothing will be gained if one uses large sample sizes in an attempt to obtain sharp discrimination between good and bad lots. Sharp discrimination is obtained only by the employment of large acceptance numbers; the size of the sample is of little importance.

One should note particularly (by examination of the tabulated values) that for small values for β the choice of sample size is quite critical. Any increase in sample size, given some value for c, allows a very considerable decrease in the required lifetesting time -- a decrease far out of proportion to the increase in sample size. Doubling the sample size may reduce the required test time to one-fourth or less of its former value, for example. This is not true, it may be noted, for large values for β. A general rule to follow if one wishes to minimize item hours of testing is to use relatively large sample sizes when the value for β is small and to use relatively small sample sizes when the value for β is large.

If in the application of these procedures the only costs of concern are those associated with the lifetesting of sample items and with the acceptance, by chance, of unsatisfactory lots, then a low acceptance number -- preferably 0 -- should be used. This practice will minimize the sample sizes and lifetesting times required. At the same time one will obtain the specified confidence, 95%, that accepted lots or populations comply with the reliable life or mean life requirement.

With this practice, however, only the consumer's risk is considered. The extent of the producer's risk -- which is the risk of rejecting acceptable lots -- is ignored. Actually, the use of low acceptance numbers maximizes this risk. The reliable life or mean life for a lot may have to be many times the life specified if there is to be a reasonable probability of acceptance. This will be true even if quite large sample sizes are employed. For Weibull sampling-inspection procedures of the form used in this report, the ability of a selected plan to discriminate sharply between acceptable and unacceptable lots depends almost entirely on the magnitude of the acceptance number; the size of the sample will make little, if any, difference. The larger the acceptance number the better the ability to discriminate, that is, the steeper the slope of the operating-characteristic curve. This is contrary to what can be expected from inspection plans of the usual attribute form for which the slope of the operating-characteristic curve depends primarily on the sample size and relatively little on the magnitude of the acceptance number.

Thus if there is concern for the producer's risk and it is important to avoid rejecting an undue number of acceptable lots, larger acceptance numbers must be used even though longer lifetesting times than for smaller numbers will be required. However, good discrimination between acceptable and unacceptable lots can be obtained only through an adequate amount of inspection and this inspection must be obtained in this way. The use of larger acceptance numbers is particularly important for small values of β. It becomes relatively less important for large β values.

To provide some guidance in the selection of acceptance numbers, a table of ratios, Table 6, has been provided. Each value tabulated is the ratio between the reliable life for which the probability of acceptance is .95 (symbolized by $L_{.95}$) and the reliable life for which the probability of acceptance is .05 (symbolized by $L_{.05}$). For each value of β, the ratios of two reliable lives are independent of their equal reliability index r and are also equal to the corresponding ratios of two mean lives. This latter life, $L_{.05}$, one should note, is the "specified reliable life" or the "specified minimum mean life" used in determining the minimum lifetesting times through the procedures and tables presented in this report. (The selection of a plan that provides a confidence level of 95% that the life requirement for the lot or population has been met provides, in effect, a consumer's risk of .05. That is, if the reliable life or mean life is precisely at the specified value, the probability of acceptance is .05.) To find the value for reliable life or mean life necessary to assure a high probability of acceptance, one need simply to multiply the specified life, $L_{.05}$, by the appropriate ratio from Table 6. This use of the ratios will be illustrated in the example that follows. It should be noted that the ratios in this table apply for all sample sizes, for all equal values of the reliability index, r, and for minimum mean life applications as well as those for reliable life.

Example (10)

A plan is required for the acceptance inspection of a mechanical component. A value for β of $1\frac{2}{3}$ and for γ of 0 can be assumed. Assurance is required that the reliable life for each lot will be at least 100 hours with the proportion surviving, or reliability index, r, being .90. A sample size of 250 items has been specified. It is also desirable that the probability of acceptance be high for lots whose reliable life is around 400 hours.

The ratio $L_{.95}/L_{.05}$ is 400/100 or 4. Reference to Table 6 indicates for $\beta = 1\frac{2}{3}$ a ratio of 4.7 for $c = 1$ and of 3.4 for $c = 2$ (these are the nearest values that can be found to the desired value of 4). With the use of 1 as the acceptance number the lifetesting time must be .36 x 100 or 36 hours (the factor .36 is obtained in the usual way from Table 4-b which gives values for $r = .90$). The reliable life for a lot must be 4.7 x 100 or 470 hours or more if the probability of acceptance is to be .95 or more. With the use of 2 as the acceptance number, the testing time must be .43 x 100 or 43 hours; the reliable life must be 3.4 x 100 or 340 hours to provide a probability of acceptance of .95. A choice can accordingly be made between these two alternatives for c.

TABLE 1 - a

Table of Values for $(t/\rho) \times 100$ $\qquad$ $r = .50$

p' (%)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010				.014	.130	.496	1.20	2.91	7.04	10.9	17.0
.012				.017	.152	.554	1.32	3.13	7.44	11.5	17.7
.015				.022	.177	.632	1.47	3.42	7.95	12.1	18.5
.020				.028	.220	.745	1.69	3.81	8.63	13.0	19.5
.025			.001	.036	.263	.860	1.90	4.20	9.27	13.8	20.5
.030			.001	.043	.299	.960	2.08	4.52	9.79	14.4	21.2
.040			.001	.058	.372	1.14	2.40	5.06	10.7	15.5	22.5
.050			.002	.072	.441	1.30	2.69	5.54	11.4	16.4	23.5
.065			.003	.094	.536	1.52	3.06	6.15	12.4	17.5	24.8
.080			.004	.115	.623	1.73	3.40	6.68	13.1	18.4	25.8
.10			.005	.144	.742	1.98	3.80	7.31	14.0	19.5	27.0
.12			.007	.173	.849	2.20	4.16	7.86	14.8	20.4	28.0
.15			.010	.216	1.01	2.52	4.65	8.59	15.9	21.6	29.3
.20		.001	.015	.289	1.25	2.99	5.37	9.64	17.3	23.2	31.0
.25		.001	.022	.361	1.47	3.42	6.01	10.5	18.5	24.5	32.5
.30		.002	.028	.433	1.68	3.82	6.58	11.3	19.5	25.6	33.7
.40		.003	.044	.579	2.10	4.54	7.61	12.7	21.3	27.6	35.7
.50		.005	.061	.723	2.49	5.19	8.50	13.9	22.8	29.2	37.3
.65		.009	.091	.941	3.01	6.08	9.70	15.5	24.7	31.1	39.3
.80		.013	.125	1.16	3.53	6.89	10.8	16.8	26.2	32.8	41.0
1.0		.021	.175	1.45	4.18	7.88	12.0	18.4	28.1	34.7	42.9
1.2	.001	.030	.230	1.74	4.78	8.80	13.2	19.8	29.7	36.3	44.5
1.5	.001	.048	.322	2.18	5.66	10.1	14.8	21.6	31.7	38.4	46.5
2.0	.002	.085	.497	2.91	7.04	12.0	17.1	24.3	34.6	41.3	49.3
2.5	.005	.133	.598	3.65	8.35	13.7	19.1	26.6	37.0	43.7	51.6
3.0	.008	.193	.921	4.39	9.61	15.3	21.0	28.6	39.2	45.8	53.5
4.0	.020	.347	1.43	5.89	12.0	18.3	24.3	32.2	42.8	49.3	56.8
5.0	.041	.548	2.01	7.40	14.1	21.0	27.2	35.3	45.8	52.1	59.4
6.5	.091	.940	3.02	9.70	17.4	24.7	31.1	39.3	49.7	55.8	62.7
8.0	.174	1.45	4.17	12.0	20.4	28.1	34.7	42.9	53.0	58.9	65.5
10	.351	2.31	5.92	15.2	24.3	32.3	39.0	47.1	56.8	62.4	68.6
12	.627	3.40	7.92	18.4	28.1	36.3	42.9	50.8	60.2	65.5	71.3
15	1.29	5.50	11.4	23.4	33.7	41.9	48.4	56.0	64.7	69.6	74.8
20	3.34	10.4	18.3	32.2	42.7	50.7	56.7	63.6	71.2	75.3	79.7
25	7.15	17.2	26.7	41.5	51.8	59.0	64.4	70.3	76.8	80.3	83.9
30	13.6	26.5	36.9	51.5	60.8	67.1	71.7	76.7	81.9	84.7	87.6
40	40.0	54.3	63.3	73.7	79.7	83.3	85.9	88.5	91.2	92.7	94.1
50	100	100	100	100	100	100	100	100	100	100	100
65	347	229	186	151	136	128	123	118	113	111	109
80	1250	539	354	232	188	166	152	140	129	124	118

TABLE 1 - b

Table of Values for (t/ρ) x 100 r = .90

p' (%)	Shape Parameter - β 1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010			.003	.095	.542	1.54	3.08	6.18	12.4	17.6	24.9
.012			.004	.114	.621	1.71	3.38	6.65	13.1	18.4	25.8
.015			.005	.142	.732	1.96	3.77	7.27	14.0	19.4	27.0
.020			.008	.190	.911	2.33	4.36	8.15	15.3	20.9	28.6
.025		.001	.012	.237	1.08	2.66	4.87	8.92	16.3	22.1	29.9
.030		.001	.015	.285	1.23	2.99	5.34	9.59	17.2	23.1	31.0
.040		.001	.023	.380	1.53	3.53	6.16	10.8	18.8	24.8	32.8
.050		.002	.033	.475	1.80	4.03	6.89	11.8	20.1	26.2	34.3
.065		.004	.048	.617	2.20	4.72	7.85	13.1	21.7	28.0	36.1
.080		.006	.066	.759	2.57	5.35	8.71	14.2	23.[illegible]	29.5	37.7
.10		.009	.092	.949	3.04	6.12	9.74	15.5	24.7	31.2	39.4
.12		.013	.121	1.14	3.49	6.82	10.7	16.7	26.1	32.7	40.9
.15		.020	.170	1.42	4.12	7.80	11.9	18.3	27.9	34.5	42.7
.20	.001	.036	.262	1.90	5.12	9.27	13.8	20.5	30.5	37.1	45.3
.25	.001	.056	.365	2.37	6.04	10.6	15.4	22.4	32.6	39.2	47.3
.30	.002	.081	.480	2.85	6.93	11.8	16.9	24.1	34.4	41.1	49.1
.40	.006	.145	.743	3.81	8.62	14.1	19.5	27.1	37.5	44.2	52.0
.50	.011	.226	1.04	4.76	10.2	16.1	21.8	29.5	40.1	46.7	54.4
.65	.024	.383	1.54	6.19	12.4	18.8	24.9	32.9	43.4	49.9	57.3
.80	.044	.581	2.04	7.62	14.5	21.3	27.6	35.7	46.2	52.5	59.8
1.0	.087	.910	2.95	9.54	17.2	24.4	30.9	39.1	49.4	55.6	62.5
1.2	.150	1.31	3.88	11.5	19.7	27.2	33.8	42.0	52.2	58.2	64.8
1.5	.295	2.06	5.43	14.3	23.3	31.2	37.9	46.0	55.8	61.5	67.8
2.0	.705	3.68	8.40	19.2	29.0	37.1	43.8	51.6	60.9	66.2	71.9
2.5	1.39	5.78	11.8	24.0	34.3	42.5	49.0	56.5	65.2	70.0	75.2
3.0	2.42	8.36	15.5	28.9	39.4	47.5	53.8	60.9	68.9	73.3	78.0
4.0	5.82	15.0	24.1	38.7	49.1	56.6	62.2	68.4	75.2	78.9	82.7
5.0	11.5	23.7	34.0	48.7	58.3	64.9	69.8	75.0	80.6	83.5	86.6
6.5	26.0	40.7	50.9	63.8	71.4	76.4	79.9	83.6	87.4	89.4	91.4
8.0	49.6	62.6	70.4	79.1	83.9	86.9	89.0	91.0	93.2	94.3	95.4
10	100	100	100	100	100	100	100	100	100	100	100
12	179	147	134	121	116	112	110	108	106	105	104
15	367	238	192	154	138	130	124	119	114	111	109
20	950	449	308	212	176	157	146	135	125	121	116
25	2030	746	451	273	213	183	165	149	135	129	122
30	3880	1150	623	339	250	208	184	163	144	136	128
40		2350	1,070	485	326	258	220	188	161	148	137
50		4330	1,690	658	410	310	257	213	176	160	146
65		9930	3,150	996	562	397	316	251	199	178	158
80			5,970	1,530	774	513	391	298	227	198	173

TABLE 1 - c

Table of Values for (t/ρ) x 100 r = .99

p' (%)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010		.010	.100	.995	3.15	6.29	9.98	15.8	25.1	31.6	39.8
.012		.014	.131	1.19	3.61	7.02	10.9	17.0	26.5	33.1	41.3
.015		.022	.182	1.49	4.26	8.02	12.2	18.6	28.3	34.9	43.1
.020	.001	.040	.281	1.99	5.30	9.50	14.1	20.8	30.9	37.6	45.6
.025	.002	.062	.392	2.49	6.26	10.9	15.8	22.8	33.0	39.7	47.8
.030	.003	.089	.516	2.99	7.18	12.2	17.3	24.5	34.9	41.6	49.5
.040	.006	.158	.794	3.98	8.91	14.5	20.0	27.5	38.0	44.7	52.5
.050	.012	.248	1.11	4.98	10.5	16.5	22.3	30.1	40.7	47.2	54.9
.065	.027	.418	1.64	6.47	12.8	19.3	25.4	33.4	44.0	50.4	57.8
.080	.050	.634	2.25	7.96	15.0	21.9	28.2	36.3	46.8	53.1	60.3
.10	.099	.990	3.14	9.95	17.7	25.0	31.5	39.7	50.0	56.2	63.0
.12	.170	1.43	4.12	11.9	20.3	27.9	34.6	42.7	52.9	58.8	65.4
.15	.333	2.23	5.76	14.9	24.0	31.9	38.6	46.7	56.5	62.2	68.4
.20	.788	3.96	8.89	19.9	29.8	38.0	44.6	52.4	61.6	66.8	72.4
.25	1.54	6.19	12.4	24.9	35.2	43.4	49.9	57.3	65.9	70.6	75.7
.30	2.66	8.91	16.3	29.9	40.4	48.4	54.6	61.7	69.58	73.9	78.5
.40	6.35	15.9	25.2	39.9	50.2	57.6	63.2	69.2	75.9	79.5	83.2
.50	12.4	24.8	35.2	49.9	59.3	65.9	70.6	75.7	81.2	84.0	87.0
.65	27.3	42.1	52.2	64.9	72.3	77.1	80.6	84.1	87.8	89.8	91.7
.80	51.0	63.8	71.4	79.9	84.5	87.4	89.4	91.4	93.5	94.6	95.6
1.0	100	100	100	100	100	100	100	100	100	100	100
1.2	173	144	132	120	115	112	110	108	106	105	104
1.5	340	226	184	150	136	128	123	118	113	111	109
2.0	812	404	285	201	169	152	142	132	123	119	115
2.5	1600	635	400	252	200	174	159	145	132	126	120
3.0	2780	919	528	303	230	195	174	156	140	132	125
4.0	6700	1,650	818	406	286	232	202	175	152	142	132
5.0		2,600	1,150	510	339	266	226	192	163	150	139
6.5		4,470	1,730	669	416	313	259	214	177	161	146
8.0		6,880	2,390	830	489	356	288	233	189	170	153
10			3,390	1,050	582	410	324	256	202	180	160
12			4,540	1,270	673	460	357	277	214	189	166
15			6,500	1,620	806	531	402	305	231	201	175
20				2,220	1,020	642	471	346	254	217	186
25				2,860	1,240	748	535	383	274	231	196
30				3,550	1,450	851	596	417	[illegible]	244	204
40				5,080	1,900	1060	713	481	325	267	219
50				6,900	2,390	1270	831	544	356	288	233
65					3,270	1630	1,020	642	403	320	253
80					4,500	2100	1,270	762	458	356	276

TABLE 2 - a

Table of Values for p' (%)   r = .50

(t/ρ) x 100	Shape Parameter - β 1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	3.17	.691	.149	.007							
.012	3.36	.757	.166	.008							
.015	3.62	.846	.195	.010							
.020	3.97	.975	.237	.014	.001						
.025	4.27	1.09	.275	.017	.001						
.030	4.53	1.19	.310	.021	.001						
.040	4.98	1.38	.376	.028	.002						
.050	5.35	1.54	.436	.035	.003						
.065	5.83	1.75	.518	.045	.004						
.080	6.23	1.94	.596	.055	.005						
.10	6.70	2.17	.691	.069	.007						
.12	7.10	2.37	.779	.083	.009						
.15	7.63	2.65	.904	.104	.012	.001					
.20	8.36	3.05	1.09	.139	.017	.002					
.25	8.98	3.41	1.27	.173	.024	.003					
.30	9.51	3.73	1.43	.202	.030	.004					
.40	10.4	4.29	1.73	.277	.044	.007	.001				
.50	11.2	4.79	2.01	.347	.059	.010	.002				
.65	12.1	5.44	2.39	.450	.084	.015	.003				
.80	12.9	6.01	2.74	.554	.110	.022	.004				
1.0	13.9	6.70	3.17	.691	.149	.032	.007				
1.2	14.7	7.31	3.57	.819	.190	.043	.010	.001			
1.5	15.7	8.14	4.13	1.03	.256	.063	.016	.002			
2.0	17.2	9.34	4.98	1.38	.376	.102	.028	.004			
2.5	18.3	10.4	5.75	1.72	.505	.148	.043	.007			
3.0	19.4	11.3	6.47	2.06	.644	.207	.062	.011			
4.0	21.1	12.9	7.79	2.74	.944	.324	.111	.022	.001		
5.0	22.5	14.4	8.98	3.41	1.27	.469	.173	.039	.003		
6.5	24.3	16.2	10.6	4.41	1.80	.726	.293	.075	.008	.001	
8.0	25.8	17.8	12.1	5.39	2.36	1.02	.443	.125	.015	.003	
10	27.5	19.7	13.9	6.70	3.17	1.48	.691	.219	.032	.007	
12	29.0	21.3	15.5	7.98	4.02	2.00	.994	.346	.059	.015	.001
15	30.8	23.5	17.8	9.88	5.37	2.99	1.55	.605	.124	.035	.006
20	33.3	26.7	21.1	12.9	7.79	4.63	2.74	1.23	.324	.111	.022
25	35.4	29.3	24.1	15.9	10.3	6.45	4.24	2.14	.680	.271	.068
30	37.1	31.6	26.6	18.8	13.0	8.90	6.05	3.36	1.25	.560	.168
40	40.0	35.5	31.4	24.2	18.5	14.0	10.5	6.78	3.22	1.76	.698
50	42.3	38.7	35.4	29.3	24.0	19.6	15.9	11.5	6.65	4.24	2.14
65	45.1	42.8	40.6	36.3	32.3	28.7	25.4	21.0	15.2	11.6	7.73
80	47.4	46.2	45.0	42.6	40.2	38.0	35.8	32.8	28.1	24.7	20.3
100	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

TABLE 2 - b

Table of Values for p' (%) r = .90

(t/ρ) x 100	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.490	.105	.022	.001							
.020	.614	.149	.036	.002							
.030	.703	.182	.047	.003							
.040	.773	.211	.057	.004							
.050	.833	.236	.066	.005							
.065	.909	.269	.079	.007							
.080	.973	.298	.091	.008							
.10	1.05	.333	.105	.011	.001						
.12	1.11	.365	.119	.013	.001						
.15	1.20	.408	.138	.016	.002						
.20	1.32	.470	.167	.021	.003						
.25	1.42	.526	.194	.026	.004						
.30	1.51	.576	.219	.032	.005						
.40	1.66	.664	.265	.042	.007	.001					
.50	1.79	.743	.308	.053	.009	.001					
.65	1.95	.846	.367	.068	.013	.002					
.80	2.09	.938	.421	.084	.017	.003					
1.0	2.24	1.05	.488	.105	.022	.005	.001				
1.2	2.38	1.15	.551	.126	.029	.007	.001				
1.5	2.57	1.28	.638	.158	.039	.009	.002				
2.0	2.82	1.48	.773	.211	.057	.015	.004				
2.5	3.03	1.65	.897	.263	.077	.022	.006	.001			
3.0	3.22	1.81	1.01	.316	.098	.030	.009	.002			
4.0	3.54	2.09	1.22	.420	.144	.049	.017	.003			
5.0	3.81	2.33	1.42	.526	.194	.071	.026	.006			
6.5	4.15	2.65	1.69	.683	.275	.111	.044	.011	.001		
8.0	4.44	2.94	1.94	.840	.363	.156	.067	.019	.002		
10	4.72	3.28	2.24	1.05	.488	.227	.105	.023	.005	.001	
12	5.07	3.58	2.53	1.26	.621	.307	.151	.052	.009	.002	
15	5.44	4.00	2.94	1.57	.837	.446	.237	.092	.019	.005	.001
20	5.96	4.60	3.54	2.09	1.22	.718	.421	.188	.049	.017	.003
25	6.42	5.13	4.10	2.60	1.65	1.04	.656	.329	.104	.041	.010
30	6.81	5.61	4.61	3.11	2.09	1.40	.944	.518	.190	.085	.026
40	7.47	6.45	5.56	4.13	3.06	2.26	1.67	1.06	.496	.270	.108
50	8.02	7.18	6.42	5.13	4.10	3.26	2.60	1.84	1.04	.657	.329
65	8.72	8.14	7.60	6.62	5.76	5.01	4.35	3.53	2.47	1.86	1.21
80	9.32	8.99	8.68	8.09	7.53	7.01	6.52	5.85	4.88	4.22	3.39
100	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
120	10.6	10.9	11.2	11.9	12.6	13.3	14.1	15.3	17.6	19.6	23.1
150	11.4	12.1	12.9	14.6	16.6	18.7	21.1	25.2	33.4	41.3	55.1
200	12.4	13.8	15.4	19.0	23.3	[illegible]	34.4	44.9	65.4	81.5	

TABLE 2 - c

Table of Values for p' (%) **r = .99**

(t/ρ) x 100	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.047	.010	.002								
.020	.059	.014	.003								
.040	.074	.020	.005								
.080	.093	.028	.009								
.10	.101	.032	.010	.001							
.20	.127	.045	.016	.002							
.40	.159	.064	.025	.004	.001						
.80	.201	.090	.040	.008	.002						
1.0	.216	.101	.047	.010	.002						
1.2	.230	.110	.053	.012	.003						
1.5	.248	.123	.061	.015	.004	.001					
2.0	.273	.142	.074	.020	.005	.001					
2.5	.294	.159	.086	.025	.007	.002					
3.0	.312	.174	.097	.030	.009	.003	.001				
4.0	.344	.201	.118	.040	.014	.005	.002				
5.0	.370	.225	.136	.050	.019	.007	.003				
6.5	.404	.256	.163	.065	.026	.010	.004	.001			
8.0	.433	.284	.187	.080	.035	.015	.006	.002			
10	.465	.318	.217	.101	.047	.022	.010	.003			
12	.495	.347	.245	.121	.060	.029	.014	.005	.001		
15	.533	.388	.285	.151	.080	.043	.023	.009	.002	.001	
20	.586	.448	.344	.201	.118	.069	.040	.018	.005	.002	
25	.631	.502	.399	.251	.158	.100	.063	.031	.010	.004	.001
30	.671	.548	.449	.302	.202	.135	.090	.049	.018	.008	.002
40	.738	.634	.545	.401	.296	.218	.161	.101	.047	.026	.010
50	.795	.709	.632	.502	.399	.316	.251	.177	.099	.063	.031
65	.867	.807	.752	.651	.565	.489	.424	.342	.239	.179	.117
80	.929	.895	.863	.801	.744	.691	.641	.574	.476	.411	.329
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
120	1.06	1.10	1.13	1.20	1.27	1.35	1.44	1.57	1.83	2.06	2.47
150	1.14	1.22	1.31	1.50	1.71	1.96	2.24	2.73	3.81	4.96	7.35
200	1.26	1.41	1.58	2.00	2.50	3.14	3.94	5.53	9.57	14.9	27.5
250	1.36	1.58	1.83	2.48	3.35	4.52	6.09	9.45	19.2	32.5	62.5
300	1.44	1.73	2.07	2.97	4.26	6.08	8.65	14.5	32.4	55.6	91.3
400	1.58	1.99	2.50	3.94	6.18	9.63	14.9	27.5	64.0	92.4	
500	1.70	2.22	2.90	4.90	8.23	13.7	22.2	43.0	88.3		
650	1.86	2.53	3.44	6.32	11.5	20.4	34.6	66.1			
800	1.99	2.80	3.94	7.73	14.9	27.5	47.4	83.8			
1000	2.14	3.13	4.56	9.56	19.5	37.3	63.4				
1500	2.45	3.82	5.93	[illegible]	31.1	60.0	89.6				

Table 3a1

Sampling Plans for $\beta = 1/3$, $r = .50$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.05	.025	.010
0	4	5	6	8	10	12	16	20	25	34	42	53	72
1	7 (.05)	8 (.03)	10 (.01)	13 (.01)	16	20	27	34	42	57	72	90	122
2	9 (.32)	11 (.16)	14 (.08)	18 (.03)	22 (.02)	28 (.01)	37	46	58	78	98	123	168
3	12 (.66)	14 (.38)	17 (.21)	23 (.08)	28 (.04)	35 (.02)	47 (.01)	58	73	98	123	156	211
4	14 (1.4)	17 (.68)	21 (.34)	27 (.14)	34 (.07)	42 (.03)	56 (.01)	70 (.01)	87	118	148	187	252
5	17 (1.8)	20 (1.0)	24 (.55)	32 (.21)	39 (.11)	49 (.05)	65 (.02)	81 (.01)	102	137	173	217	293
6	19 (2.7)	23 (1.3)	28 (.70)	36 (.29)	45 (.14)	56 (.07)	74 (.03)	92 (.01)	115 (.01)	157	197	247	332
7	21 (3.5)	26 (1.7)	31 (.90)	41 (.36)	50 (.19)	62 (.09)	82 (.04)	103 (.02)	129 (.01)	176 (.01)	220	276	371
8	24 (4.1)	28 (2.3)	34 (1.1)	45 (.47)	56 (.22)	69 (.11)	91 (.05)	113 (.02)	143 (.01)	194	243	304	410
9	26 (4.9)	31 (2.7)	38 (1.3)	49 (.50)	61 (.25)	75 (.13)	100 (.05)	124 (.03)	158 (.01)	212	266	333	448
10	30 (4.9)	35 (2.7)	43 (1.3)	56 (.50)	68 (.25)	84 (.13)	111 (.05)	138 (.03)	172 (.01)	230	288	361	486

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3a2

Sampling Plans for $\beta = 1/2$, $r = .50$

c	n — $(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less): 100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	4 (.03)	5 (.02)	7 (.01)	11	15	21	34	47	67	105	150	212	334
1	7 (.62)	9 (.37)	12 (.20)	19 (.07)	26 (.04)	36 (.02)	57 (.01)	80	113	180	253	357	563
2	9 (2.2)	12 (1.1)	17 (.55)	26 (.22)	36 (.11)	50 (.06)	78 (.02)	110 (.01)	156	245	346	488	770
3	12 (3.6)	16 (1.9)	21 (1.0)	32 (.42)	45 (.21)	63 (.10)	98 (.04)	139 (.02)	196 (.01)	308	434	613	967
4	14 (5.8)	19 (2.8)	26 (1.4)	39 (.59)	54 (.30)	75 (.15)	118 (.06)	167 (.03)	234 (.01)	368	519	733	1160
5	17 (6.9)	22 (3.7)	30 (1.9)	45 (.79)	63 (.39)	87 (.20)	137 (.08)	194 (.04)	272 (.02)	427 (.01)	602	851	1340
6	19 (9.1)	25 (4.7)	34 (2.3)	51 (1.0)	71 (.49)	99 (.25)	157 (.09)	220 (.05)	309 (.02)	485 (.01)	684	966	1530
7	21 (11)	28 (5.6)	38 (2.8)	58 (1.1)	80 (.56)	111 (.29)	176 (.11)	246 (.05)	345 (.03)	542 (.01)	764	1080	1700
8	24 (12)	31 (6.5)	42 (3.2)	64 (1.3)	88 (.65)	123 (.33)	194 (.12)	271 (.06)	381 (.03)	599 (.01)	844	1190	1880
9	26 (13)	34 (7.3)	46 (3.6)	70 (1.4)	96 (.73)	134 (.37)	212 (.14)	297 (.07)	417 (.03)	655 (.01)	923	1300	2060
10	30 (13)	38 (7.3)	51 (3.6)	77 (1.4)	107 (.73)	148 (.37)	230 (.15)	322 (.08)	452 (.04)	711 (.01)	1000	1410	2230

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3a3

Sampling Plans for β = 2/3, r = .50

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.05	.025	.010
0	4 (.25)	6 (.13)	9 (.07)	16 (.03)	25 (.02)	39 (.01)	72	114	182	334	529	838	1,550
1	7 (2.2)	10 (1.2)	15 (.68)	27 (.27)	42 (.13)	66 (.07)	122 (.03)	194 (.01)	307 (.01)	563	893	1,420	2,610
2	9 (5.6)	14 (2.7)	21 (1.5)	37 (.60)	58 (.30)	91 (.15)	168 (.06)	265 (.03)	419 (.01)	771	1,220	1,940	3,580
3	12 (8.2)	17 (4.5)	26 (2.3)	47 (.93)	73 (.47)	115 (.2[illegible])	211 (.09)	333 (.04)	526 (.02)	967 (.01)	1,530	2,430	4,490
4	14 (11)	21 (5.8)	31 (3.1)	56 (1.2)	87 (.62)	137 (.31)	253 (.12)	398 (.06)	630 (.03)	1,160 (.01)	1,840	2,910	5,370
5	17 (13)	24 (7.4)	37 (3.6)	65 (1.5)	102 (.75)	162 (.36)	293 (.15)	462 (.07)	731 (.03)	1,340 (.01)	2,130 (.01)	3,380	6,230
6	19 (16)	28 (8.4)	42 (4.1)	74 (1.7)	115 (.89)	184 (.43)	333 (.17)	524 (.09)	830 (.04)	1,520 (.02)	2,420 (.01)	3,830	7,070
7	21 (19)	31 (9.5)	47 (4.7)	82 (2.0)	129 (1.0)	205 (.48)	372 (.19)	586 (.10)	927 (.05)	1,710 (.02)	2,700 (.01)	4,280	7,900
8	24 (20)	34 (10)	52 (5.2)	91 (2.2)	143 (1.1)	226 (.54)	410 (.21)	647 (.10)	1,030 (.05)	1,880 (.02)	2,980 (.01)	4,730	8,720
9	26 (21)	38 (10)	57 (5.4)	100 (2.3)	159 (1.1)	248 (.58)	448 (.23)	707 (.11)	1,120 (.06)	2,060 (.02)	3,260 (.01)	5,170 (.01)	9,540
10	30 (22)	44 (10)	64 (5.5)	111 (2.3)	172 (1.2)	268 (.63)	487 (.25)	767 (.12)	1,220 (.06)	2,230 (.02)	3,540 (.01)	5,610 (.01)	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a4

Sampling Plans for $\beta = 1$, $r = .50$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	15	10	5.0	2.5	1.5	1.0	.50	.25	.15	.10
0	4 (1.8)	7 (1.0)	14 (.53)	23 (.33)	34 (.22)	67 (.11)	133 (.06)	224 (.03)	334 (.02)	664 (.01)	1,330	2,210	3,340
1	7 (7.7)	12 (4.4)	23 (2.3)	38 (1.3)	57 (.90)	113 (.45)	226 (.22)	378 (.13)	563 (.09)	1120 (.04)	2250 (.02)	3740 (.01)	5640 (.01)
2	9 (14)	17 (7.3)	32 (3.7)	53 (2.3)	78 (1.5)	156 (.75)	309 (.38)	517 (.23)	770 (.15)	1530 (.07)	3080 (.04)	5120 (.02)	7710 (.01)
3	12 (18)	21 (10)	40 (5.1)	66 (3.0)	98 (2.0)	196 (1.0)	388 (.51)	649 (.30)	967 (.20)	1930 (.10)	3860 (.05)	6420 (.03)	9680 (.02)
4	14 (23)	26 (12)	49 (6.0)	79 (3.7)	118 (2.4)	234 (1.2)	465 (.61)	776 (.36)	1160 (.24)	2300 (.12)	4620 (.06)	7690 (.04)	
5	17 (26)	30 (13)	56 (6.9)	92 (4.1)	137 (2.8)	272 (1.4)	539 (.70)	900 (.42)	1340 (.28)	2670 (.14)	5360 (.07)	8920 (.04)	
6	19 (30)	34 (15)	64 (7.6)	105 (4.6)	157 (3.0)	309 (1.5)	612 (.77)	1020 (.46)	1530 (.31)	3040 (.15)	6090 (.08)		
7	21 (33)	38 (16)	72 (8.3)	117 (5.0)	176 (3.2)	345 (1.6)	684 (.84)	1140 (.50)	1700 (.34)	3390 (.17)	6800 (.08)		
8	24 (34)	42 (18)	79 (8.9)	129 (5.3)	194 (3.5)	381 (1.8)	755 (.90)	1260 (.54)	1880 (.36)	3740 (.18)	7510 (.09)		
9	26 (35)	46 (18)	87 (9/3)	142 (5.6)	213 (3.7)	417 (1.9)	827 (.94)	1,380 (.57)	2,060 (.38)	4,100 (.19)	8,210 (.09)		
10	30 (36)	53 (18)	97 (9.7)	156 (5.7)	230 (3.9)	452 (1.9)	896 (.98)	1,500 (.59)	2,230 (.39)	4,440 (.20)	8,910 (.10)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a5

Sampling Plans for $\beta = 1\ 1/3$, $r = .50$

c	n (t/ρ) x 100 Ratio for which P(A) = .10 (or less) 100	50	40	25	15	10	8.0	5.0	4.0	2.5	1.5	1.0	.50
0	4 (4.9)	9 (2.7)	12 (2.2)	22 (1.4)	42 (.86)	72 (.57)	97 (.46)	181 (.28)	242 (.23)	452 (.14)	900 (.08)	1540 (.06)	3840 (.03)
1	7 (15)	15 (8.0)	20 (6.4)	37 (4.0)	71 (2.5)	122 (1.6)	165 (1.3)	306 (.82)	409 (.66)	763 (.41)	1520 (.24)	2600 (.16)	6480 (.08)
2	9 (24)	21 (12)	27 (9.8)	50 (6.0)	98 (3.6)	168 (2.4)	226 (1.9)	419 (1.2)	560 (1.0)	1050 (.61)	2080 (.36)	3550 (.24)	8870 (.12)
3	12 (29)	26 (15)	35 (12)	63 (7.5)	123 (4.5)	211 (3.0)	283 (2.4)	526 (1.5)	703 (1.2)	1310 (.76)	2610 (.45)	4460 (.30)	
4	14 (34)	32 (17)	42 (13)	76 (8.6)	147 (5.2)	252 (3.4)	339 (2.7)	629 (1.7)	841 (1.4)	1570 (.88)	3120 (.52)	5330 (.35)	
5	17 (36)	37 (19)	48 (15)	88 (9.5)	173 (5.6)	293 (3.8)	393 (3.0)	730 (1.9)	976 (1.5)	1820 (.97)	3620 (.58)	6180 (.38)	
6	19 (41)	42 (20)	55 (16)	100 (10)	196 (6.1)	332 (4.1)	446 (3.3)	829 (2.1)	1110 (1.6)	2070 (1.0)	4110 (.62)	7020 (.41)	
7	21 (44)	47 (21)	61 (17)	112 (11)	219 (6.6)	371 (4.3)	499 (3.5)	927 (2.2)	1240 (1.7)	2310 (1.1)	4600 (.66)	7850 (.45)	
8	24 (45)	52 (23)	68 (18)	124 (11)	242 (6.8)	410 (4.6)	550 (3.6)	1020 (2.3)	1370 (1.8)	2550 (1.1)	5080 (.70)	8660 (.46)	
9	26 (46)	57 (24)	74 (19)	135 (12)	265 (7.1)	448 (4.8)	602 (3.8)	1120 (2.4)	1500 (1.9)	2790 (1.2)	5550 (.72)	9470 (.48)	
10	29 (46)	63 (24)	82 (19)	150 (12)	287 (7.4)	486 (5.0)	553 (4.0)	1210 (2.5)	1620 (2.0)	3020 (1.2)	6020 (.75)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a6

Sampling Plans for $\beta = 1\ 2/3$, $r = .50$

c	n												
	$(t/\rho) \times 100$ Ratio for which P(A) = .10 or less												
	100	80	50	40	25	15	10	8.0	5.0	4.0	2.5	1.5	1.0
0	4 (9.0)	5 (7.9)	11 (4.9)	16 (3.9)	35 (2.4)	76 (1.5)	156 (1.0)	224 (.81)	480 (.51)	698 (.41)	1540 (.25)	3600 (.15)	7090 (.10)
1	7 (21)	9 (18)	19 (11)	27 (9.2)	59 (5.8)	129 (3.6)	263 (2.3)	378 (1.9)	810 (1.2)	1180 (.95)	2590 (.60)	6080 (.36)	
2	9 (31)	13 (24)	26 (16)	37 (12)	81 (7.8)	178 (4.8)	360 (3.2)	517 (2.6)	1110 (1.6)	1610 (1.3)	3550 (.81)	8320 (.49)	
3	12 (36)	16 (30)	33 (18)	46 (15)	102 (9.3)	223 (5.8)	451 (3.8)	650 (3.0)	1390 (1.9)	2030 (1.5)	4460 (.97)		
4	14 (42)	19 (34)	39 (21)	55 (17)	122 (10)	267 (6.4)	540 (4.2)	776 (3.4)	1670 (2.1)	2420 (1.7)	5330 (1.1)		
5	17 (44)	22 (37)	45 (23)	64 (18)	142 (11)	310 (7.0)	627 (4.6)	900 (3.7)	1930 (2.3)	2810 (1.8)	6180 (1.1)		
6	19 (48)	26 (38)	52 (24)	73 (19)	163 (12)	352 (7.4)	711 (4.9)	1020 (3.9)	2200 (2.5)	3190 (2.0)	7020 (1.2)		
7	21 (51)	29 (40)	58 (25)	82 (20)	182 (12)	394 (7.8)	795 (5.1)	1140 (4.1)	2450 (2.6)	3570 (2.1)	7850 (1.3)		
8	24 (52)	32 (42)	65 (26)	90 (21)	201 (13)	434 (8.2)	878 (5.3)	1260 (4.3)	2710 (2.7)	3940 (2.1)	8660 (1.3)		
9	26 (53)	35 (44)	70 (27)	99 (22)	220 (13)	475 (8.4)	960 (5.5)	1380 (4.4)	2960 (2.8)	4310 (2.2)	9470 (1.4)		
10	29 (54)	39 (44)	78 (27)	110 (22)	239 (14)	515 (8.7)	1040 (5.7)	1500 (4.6)	3210 (2.9)	4670 (2.3)			

$(t/\rho) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

Table 3a7

Sampling Plans for $\beta = 2$, $r = .50$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	50	40	25	15	10	8.0	5.0	4.0	3.0	2.5	2.0
0	4 (13)	6 (11)	14 (7.2)	21 (5.9)	54 (3.6)	147 (2.2)	334 (1.5)	520 (1.2)	1330 (.74)	2080 (.60)	3710 (.45)	5360 (.37)	8220 (.28)
1	7 (28)	10 (23)	23 (15)	36 (12)	91 (7.5)	251 (4.5)	563 (3.0)	878 (2.4)	2250 (1.5)	3500 (1.2)	6280 (.90)	9050 (.75)	
2	9 (38)	14 (30)	32 (19)	49 (15)	124 (9.8)	343 (5.8)	770 (4.1)	1200 (3.1)	3080 (1.9)	4800 (1.5)	8590 (1.1)		
3	12 (43)	17 (35)	40 (22)	62 (18)	158 (11)	431 (6.7)	967 (4.5)	1510 (3.6)	3860 (2.2)	6020 (1.8)			
4	14 (48)	21 (38)	49 (24)	74 (20)	189 (13)	516 (7.4)	1160 (5.0)	1810 (3.9)	4620 (2.4)	7200 (1.9)			
5	17 (50)	24 (41)	56 (26)	86 (21)	219 (13)	598 (7.9)	1340 (5.3)	2090 (4.2)	5360 (2.6)	8360 (2.1)			
6	19 (54)	27 (44)	64 (27)	98 (22)	248 (14)	679 (8.4)	1520 (5.5)	2380 (4.4)	6090 (2.7)	9490 (2.2)			
7	21 (57)	31 (45)	72 (28)	110 (23)	278 (14)	759 (8.7)	1700 (5.8)	2660 (4.6)	6800 (2.8)				
8	24 (58)	34 (47)	79 (30)	121 (24)	306 (15)	838 (9.0)	1880 (6.0)	2930 (4.8)	7510 (2.9)				
9	26 (58)	37 (47)	87 (31)	132 (25)	335 (15)	917 (9.2)	2,060 (6.1)	3,210 (4.9)	8,210 (3.0)				
10	30 (58)	43 (47)	97 (31)	147 (25)	363 (16)	994 (9.4)	2230 (6.3)	3480 (5.0)	8910 (3.1)				

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a8

Sampling Plans for β = 2 1/2, r = .50

c	n: (t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	65	50	40	30	25	20	15	12	10	8.0	5.0
0	4 (20)	6 (17)	10 (14)	19 (10)	33 (8.6)	68 (6.5)	107 (6.5)	187 (4.3)	378 (3.3)	662 (2.6)	1000 (2.2)	1840 (1.7)	5910 (1.1)
1	7 (36)	11 (29)	17 (24)	33 (18)	56 (15)	115 (11)	182 (9.4)	316 (7.6)	638 (5.7)	1120 (4.6)	1690 (3.8)	3110 (3.0)	9980 (1.9)
2	9 (46)	15 (27)	24 (30)	45 (23)	77 (18)	158 (14)	249 (11)	433 (9.3)	872 (7.1)	1530 (5.6)	2320 (4.8)	4260 (3.7)	
3	12 (51)	19 (41)	30 (34)	56 (26)	97 (20)	199 (15)	312 (13)	543 (10)	1100 (7.9)	1920 (6.3)	2910 (5.3)	5350 (4.2)	
4	14 (56)	23 (44)	36 (37)	68 (28)	116 (22)	238 (17)	374 (14)	650 (11)	1310 (8.5)	2300 (6.8)	3480 (5.8)	6400 (4.5)	
5	17 (58)	26 (48)	42 (38)	79 (29)	135 (23)	276 (18)	433 (15)	754 (12)	1520 (9.0)	2670 (7.2)	4030 (6.1)	7420 (4.8)	
6	19 (61)	30 (50)	48 (40)	89 (31)	155 (24)	313 (18)	492 (15)	856 (12)	1730 (9.3)	3030 (7.5)	4580 (6.3)	8430 (5.0)	
7	21 (63)	34 (51)	54 (41)	100 (32)	174 (25)	350 (19)	550 (16)	957 (12)	1930 (9.7)	3380 (7.7)	5120 (6.6)	9420 (5.1)	
8	24 (64)	37 (52)	60 (43)	110 (32)	192 (26)	387 (19)	607 (16)	1060 (13)	2130 (10)	3730 (7.9)	5650 (6.7)		
9	26 (65)	41 (53)	65 (43)	121 (33)	210 (26)	423 (20)	664 (16)	1160 (13)	2330 (10)	4080 (8.1)	6180 (6.9)		
10	29 (65)	46 (53)	72 (43)	134 (34)	227 (27)	459 (20)	720 (17)	1250 (13)	2530 (10)	4430 (8.2)	6670 (7.0)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b1

Sampling Plans for $\beta = 1/3$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	22	28	35	48	60	76	104	129	163	220	277	348	470
1	38 (.08)	48 (.03)	60 (.02)	82 (.01)	101	129	174	217	274	370	467	587	794
2	52 (.35)	65 (.18)	82 (.08)	112 (.03)	139 (.02)	176 (.01)	237	297	375	507	639	804	1,090
3	65 (.84)	82 (.41)	103 (.21)	141 (.08)	175 (.04)	220 (.02)	298 (.01)	373	470	636	802	1,010	1,360
4	78 (1.4)	98 (.73)	123 (.37)	169 (.14)	210 (.07)	264 (.03)	357 (.01)	446 (.01)	563	761	960	1,210	1,630
5	91 (2.2)	114 (1.1)	143 (.54)	196 (.20)	243 (.10)	306 (.05)	414 (.02)	518 (.01)	653	883	1,110	1,400	1,890
6	103 (2.9)	130 (1.4)	164 (.71)	223 (.27)	276 (.14)	347 (.07)	470 (.03)	588 (.01)	741 (.01)	1,000	1,260	1,590	2,150
7	115 (3.8)	145 (1.9)	184 (.89)	250 (.35)	309 (.18)	389 (.09)	526 (.03)	658 (.02)	829 (.01)	1120	1410	1780	2400
8	127 (4.7)	162 (2.1)	202 (1.0)	275 (.43)	341 (.22)	429 (.11)	580 (.04)	726 (.02)	915 (.01)	1,240	1,560	1,960	2,650
9	139 (5.5)	177 (2.5)	221 (1.2)	301 (.51)	373 (.26)	469 (.13)	634 (.05)	794 (.03)	1,000 (.01)	1,350	1,710	2,150	2,900
10	150 (5.8)	193 (2.9)	240 (1.4)	327 (.58)	405 (.29)	509 (.15)	688 (.06)	861 (.03)	1090 (.01)	1470	1850	2330	3150

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b2

Sampling Plans for $\beta = 1/2$, $r = .90$

c	n: (t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	22 (.05)	31 (.02)	45 (.01)	69	98	138	219	307	435	687	980	1400	2190
1	38 (.80)	53 (.40)	75 (.20)	117 (.08)	167 (.04)	236 (.02)	370 (.01)	519	734	1160	1660	2360	3710
2	52 (2.3)	73 (1.2)	102 (.59)	162 (.23)	228 (.11)	323 (.06)	507 (.02)	710 (.01)	1000	1590	2270	3230	5070
3	65 (3.9)	92 (2.0)	129 (1.0)	204 (.40)	287 (.20)	405 (.10)	636 (.04)	891 (.02)	1260 (.01)	1990	2840	4050	6360
4	78 (6.0)	110 (3.0)	156 (1.4)	244 (.59)	343 (.30)	484 (.15)	761 (.06)	1070 (.03)	1510 (.01)	2390	3400	4850	7610
5	91 (8.0)	127 (4.0)	181 (1.9)	283 (.76)	398 (.38)	562 (.19)	883 (.08)	1240 (.04)	1750 (.02)	2770 (.01)	3950	5620	8830
6	103 (9.6)	145 (4.8)	205 (2.3)	321 (.94)	452 (.47)	638 (.24)	1000 (.09)	1400 (.04)	1990 (.02)	3140 (.01)	4480	6380	
7	115 (11)	164 (5.4)	229 (2.8)	359 (1.1)	505 (.55)	713 (.28)	1120 (.11)	1570 (.06)	2220 (.03)	3510 (.01)	5010	7130	
8	127 (13)	181 (6.2)	253 (3.1)	396 (1.3)	558 (.63)	787 (.32)	1240 (.13)	1730 (.06)	2450 (.03)	3880 (.01)	5530	7870	
9	139 (15)	198 (6.9)	277 (3.4)	433 (1.4)	610 (.71)	861 (.35)	1350 (.14)	1900 (.07)	2680 (.04)	4240 (.01)	6050 (.01)	8610	
10	154 (15)	215 (7.6)	300 (3.8)	470 (1.5)	661 (.77)	934 (.38)	1470 (.16)	2060 (.08)	2910 (.04)	4600 (.01)	6560 (.01)	9340	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b3

Sampling Plans for β = 2/3 r = .90

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	100	50	25	15	10	5.0	2.5	1.0	.50	.25	.10	.05
0	14	22	35	55	78	102	162	259	470	743	1180	2170	3440
	(.63)	(.33)	(.16)	(.08)	(.05)	(.03)	(.02)	(.01)					
1	24	38	59	94	131	173	274	437	794	1260	2000	3670	5810
	(5.4)	(2.7)	(1.4)	(.68)	(.41)	(.27)	(.13)	(.07)	(.03)	(.01)			
2	33	52	82	128	181	237	375	598	1090	1720	2730	5020	7940
	(11)	(5.9)	(2.9)	(1.5)	(.87)	(.59)	(.29)	(.14)	(.06)	(.03)	(.01)		
3	42	65	103	163	227	297	471	751	1360	2160	3430	6300	9970
	(18)	(8.8)	(4.6)	(2.2)	(1.4)	(.90)	(.46)	(.22)	(.09)	(.04)	(.02)	(.01)	
4	50	78	123	195	272	355	563	898	1630	2580	4100	7540	
	(24)	(12)	(6.1)	(3.0)	(1.8)	(1.2)	(.60)	(.30)	(.12)	(.06)	(.03)	(.01)	
5	58	91	143	226	315	412	653	1040	1890	2990	4760	8750	
	(30)	(15)	(7.4)	(3.6)	(2.2)	(1.5)	(.74)	(.36)	(.14)	(.07)	(.04)	(.01)	
6	66	103	164	257	358	468	742	1180	2150	3400	5400	9930	
	(35)	(17)	(8.4)	(4.3)	(2.6)	(1.7)	(.86)	(.42)	(.18)	(.09)	(.04)	(.02)	
7	74	115	183	287	400	523	829	1320	2400	3800	6040		
	(39)	(20)	(9.5)	(4.8)	(2.9)	(1.9)	(.96)	(.48)	(.20)	(.10)	(.05)		
8	82	127	202	317	442	578	915	1460	2650	4190	6660		
	(43)	(22)	(10)	(5.3)	(3.2)	(2.1)	(1.0)	(.52)	(.21)	(.10)	(.05)		
9	90	139	221	347	483	632	1000	1600	2900	4580	7290		
	(47)	(23)	(11)	(5.7)	(3.5)	(2.3)	(1.1)	(.58)	(.24)	(.12)	(.06)		
10	100	154	240	376	524	685	1090	1730	3150	4970	7900		
	(47)	(24)	(12)	(6.2)	(3.8)	(2.5)	(1.2)	(.62)	(.25)	(.12)	(.06)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b4

Sampling Plans for $\beta = 1$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	100	80	50	25	15	10	8.0	5.0	2.5	1.5	1.0
0	11 (4.4)	15 (3.3)	22 (2.2)	28 (1.7)	44 (1.1)	88 (.55)	146 (.33)	219 (.22)	274 (.18)	438 (.11)	876 (.05)	1,460 (.03)	2,190 (.02)
1	19 (18)	26 (13)	38 (9.0)	47 (7.2)	75 (4.5)	148 (2.2)	248 (1.3)	370 (.92)	463 (.73)	740 (.45)	1,480 (.22)	2,460 (.13)	3,710 (.09)
2	27 (29)	35 (22)	52 (15)	65 (12)	102 (7.7)	205 (3.8)	339 (2.2)	507 (1.5)	634 ([illegible].2)	1,010 (.77)	2,020 (.38)	3,370 (.22)	5,070 (.15)
3	34 (39)	44 (30)	65 (20)	81 (16)	129 (10)	257 (5.0)	426 (3.0)	636 (2.0)	795 (1.6)	1,270 (1.0)	2,540 (.50)	4,230 (.30)	6,360 (.20)
4	40 (49)	53 (36)	78 (24)	97 (19)	156 (12)	307 (6.0)	509 (3.6)	761 (2.4)	952 (1.9)	1,520 (1.2)	3,040 (.61)	5,060 (.36)	7,610 (.24)
5	47 (55)	62 (41)	91 (29)	113 (22)	181 (14)	357 (7.0)	591 (4.2)	883 (2.8)	1,100 (2.2)	1,760 (1.4)	3,530 (.70)	5,870 (.41)	8,830 (.28)
6	53 (61)	70 (46)	103 (30)	128 (25)	205 (15)	405 (7.7)	671 (4.6)	1,000 (3.1)	1,250 (2.5)	2,000 (1.5)	4,000 (.78)	6,670 (.46)	
7	60 (66)	78 (51)	115 (33)	143 (27)	229 (16)	453 (8.4)	750 (5.0)	1,120 (3.3)	1,400 (2.7)	2,240 (1.6)	4,480 (.85)	7,450 (.50)	
8	67 (70)	87 (53)	127 (36)	161 (28)	253 (18)	500 (9.0)	827 (5.4)	1,240 (3.6)	1,550 (2.9)	2,470 (1.8)	4,940 (.90)	8,220 (.54)	
9	72 (75)	95 (57)	139 (38)	176 (30)	277 (19)	547 (9.5)	905 (5.7)	1,350 (3.8)	1,690 (3.0)	2,700 (1.9)	5,400 (.95)	8,990 (.57)	
10	80 (76)	106 (58)	155 (39)	191 (31)	301 (20)	593 (10)	982 (6.0)	1,470 (4.0)	1,840 (3.1)	2,930 (2.0)	5,860 (1.0)	9,760 (.6)	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b5

Sampling Plans for β = 1 1/3, r = .90

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	100	80	50	40	25	15	10	8.0	5.0	4.0	2.5
0	9 (11)	13 (8.3)	22 (5.6)	30 (4.5)	55 (2.8)	74 (2.3)	138 (1.4)	274 (.87)	461 (.58)	622 (.45)	1150 (.29)	1540 (.23)	2950 (.14)
1	16 (31)	22 (25)	38 (16)	51 (13)	94 (8.0)	126 (6.5)	236 (4.1)	463 (2.5)	778 (1.7)	1050 (1.3)	1950 (.83)	2590 (.69)	4990 (.45)
2	21 (49)	31 (36)	52 (24)	69 (19)	128 (12)	174 (9.5)	323 (6.0)	634 (3.6)	1070 (2.5)	1440 (2.0)	2660 (1.2)	3550 (1.0)	6820 (.62)
3	27 (59)	39 (45)	65 (30)	87 (24)	163 (15)	218 (12)	405 (7.4)	795 (4.5)	1340 (3.0)	1810 (2.4)	3340 (1.5)	4450 (1.2)	8570 (.76)
4	33 (67)	46 (52)	78 (34)	105 (27)	195 (17)	261 (13)	485 (8.5)	952 (5.1)	1600 (3.5)	2160 (2.8)	4000 (1.7)	5330 (1.4)	
5	38 (75)	54 (57)	91 (38)	121 (29)	226 (19)	303 (15)	562 (9.5)	1100 (5.7)	1860 (3.9)	2510 (3.1)	4640 (1.9)	6180 (1.6)	
6	43 (81)	61 (61)	103 (41)	138 (33)	257 (20)	344 (16)	638 (10)	1250 (6.2)	2110 (4.2)	2850 (3.3)	5270 (2.1)	7020 (1.7)	
7	48 (87)	69 (66)	115 (44)	156 (34)	287 (22)	385 (17)	713 (11)	1400 (6.5)	2360 (4.4)	3180 (3.5)	5890 (2.2)	7850 (1.8)	
8	54 (90)	76 (69)	127 (46)	173 (36)	317 (23)	425 (18)	787 (11)	1550 (6.9)	2600 (4.7)	3510 (3.7)	6500 (2.3)	8660 (1.9)	
9	59 (94)	83 (72)	139 (49)	189 (37)	347 (24)	464 (19)	861 (12)	1690 (7.2)	2840 (4.9)	3840 (3.9)	7110 (2.4)	9470 (1.9)	
10	66 (95)	93 (72)	154 (49)	205 (39)	376 (25)	504 (19)	934 (12)	1840 (7.4)	3080 (5.0)	4170 (4.0)	7710 (2.5)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b6

Sampling Plans for $\beta = 1\ 2/3$, $r = .90$

c	n (t/ρ) x 100 Ratio for which P(A) = .10 (or less) 200	150	100	80	65	50	40	25	15	10	8.0	5.0	4.0
0	7 (20)	12 (14)	22 (10)	32 (8.1)	45 (6.6)	70 (5.0)	101 (4.1)	221 (2.5)	518 (1.5)	1010 (1.0)	1470 (.85)	3200 (.50)	4650 (.40)
1	13 (45)	20 (35)	38 (23)	54 (19)	77 (15)	118 (12)	172 (9.4)	374 (5.9)	874 (3.6)	1710 (2.4)	2480 (1.8)	5400 (1.2)	7860 (.95)
2	17 (65)	27 (48)	52 (32)	75 (26)	105 (21)	163 (16)	235 (13)	512 (8.1)	1200 (4.8)	2330 (3.2)	3390 (2.6)	7390 (1.6)	
3	22 (75)	34 (57)	65 (38)	94 (30)	132 (25)	205 (19)	296 (15)	642 (9.6)	1500 (5.7)	2930 (3.9)	4260 (3.1)	9280 (1.9)	
4	26 (85)	41 (64)	78 (43)	112 (34)	160 (28)	245 (21)	354 (17)	769 (11)	1800 (6.5)	3510 (4.3)	5090 (3.4)		
5	31 (90)	48 (69)	91 (46)	131 (37)	185 (30)	285 (23)	410 (18)	892 (11)	2090 (7.0)	4070 (4.6)	5910 (3.7)		
6	35 (97)	54 (74)	103 (48)	148 (39)	210 (32)	323 (24)	466 (19)	1010 (12)	2370 (7.4)	4620 (4.9)	6710 (3.9)		
7	39 (105)	61 (77)	115 (52)	168 (41)	235 (33)	361 (26)	521 (20)	1130 (13)	2650 (7.8)	5160 (5.2)	7500 (4.1)		
8	44 (108)	68 (80)	127 (54)	185 (43)	259 (35)	398 (27)	575 (21)	1250 (13)	2920 (8.1)	5700 (5.4)	8280 (4.3)		
9	48 (110)	74 (83)	139 (56)	203 (44)	284 (36)	439 (27)	629 (22)	1370 (14)	3190 (8.4)	6230 (5.6)	9050 (4.5)		
10	53 (110)	82 (83)	154 (57)	220 (45)	308 (37)	473 (28)	682 (23)	1480 (14)	3460 (8.7)	6760 (5.7)	9820 (4.6)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b7

Sampling Plans for $\beta = 2$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	120	100	80	65	50	40	25	20	15	10	8.0
0	6 (28)	10 (22)	15 (18)	22 (14)	34 (12)	52 (9.6)	88 (7.4)	137 (5.9)	349 (3.7)	548 (2.9)	960 (2.2)	2170 (1.5)	3390 (1.2)
1	10 (60)	17 (45)	26 (36)	38 (30)	59 (24)	88 (19)	148 (15)	233 (12)	589 (7.5)	926 (5.9)	1620 (4.5)	3670 (3.0)	5720 (2.4)
2	14 (77)	24 (58)	36 (47)	52 (39)	80 (31)	121 (25)	205 (19)	319 (15)	806 (9.7)	1270 (7.8)	2220 (5.9)	5020 (3.8)	7830 (3.0)
3	18 (88)	30 (68)	46 (54)	65 (45)	101 (36)	154 (29)	257 (22)	400 (18)	1010 (11)	1590 (9.0)	2780 (6.7)	6300 (4.5)	9830 (3.6)
4	21 (99)	36 (74)	55 (59)	78 (50)	121 (39)	184 (31)	307 (24)	479 (20)	1210 (12)	1900 (9.9)	3330 (7.4)	7540 (4.9)	
5	25 (105)	42 (79)	64 (63)	91 (53)	140 (41)	213 (34)	357 (26)	555 (21)	1410 (13)	2210 (10)	3870 (7.9)	8750 (5.3)	
6	29 (110)	48 (83)	73 (66)	103 (56)	161 (44)	242 (36)	405 (28)	631 (22)	1600 (14)	2510 (11)	4390 (8.4)	9930 (5.5)	
7	32 (115)	54 (86)	81 (70)	115 (58)	181 (46)	271 (37)	453 (29)	705 (23)	1780 (14)	2800 (11)	4900 (8.7)		
8	36 (118)	60 (89)	90 (72)	127 (60)	199 (47)	299 (38)	500 (30)	778 (24)	1970 (15)	3090 (12)	5410 (9.0)		
9	39 (123)	65 (91)	98 (74)	139 (62)	218 (49)	327 (40)	546 (31)	851 (24)	2150 (15)	3380 (12)	5920 (9.3)		
10	45 (123)	73 (91)	109 (74)	154 (62)	236 (50)	354 (41)	593 (31)	923 (25)	2340 (16)	3670 (12)	6240 (9.6)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b8

Sampling Plans for β = 2 1/2, r = .90

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	120	100	80	65	50	40	30	25	20	15	12
0	4 (43)	8 (32)	14 (26)	22 (21)	39 (17)	64 (14)	124 (11)	217 (8.7)	439 (6.5)	700 (5.3)	1210 (4.3)	2480 (3.2)	4300 (2.6)
1	8 (72)	14 (57)	24 (46)	38 (38)	65 (30)	109 (24)	211 (19)	367 (15)	741 (11)	1180 (9.5)	2050 (7.6)	4180 (5.7)	7270 (4.5)
2	10 (94)	20 (70)	33 (57)	52 (47)	90 (37)	149 (30)	289 (23)	502 (19)	1010 (14)	1610 (12)	2800 (9.4)	5720 (7.0)	9950 (5.6)
3	13 (105)	25 (79)	42 (63)	65 (53)	113 (42)	189 (34)	363 (26)	630 (21)	1270 (14)	2030 (13)	3520 (10)	7180 (7.9)	
4	16 (113)	30 (85)	51 (68)	78 (57)	135 (45)	226 (37)	434 (28)	754 (23)	1520 (17)	2420 (14)	4210 (11)	8600 (8.6)	
5	19 (118)	35 (89)	59 (72)	91 (60)	159 (47)	263 (38)	504 (30)	875 (24)	1770 (18)	2810 (15)	4880 (12)	9970 (9.0)	
6	21 (125)	40 (93)	67 (75)	103 (63)	180 (50)	298 (40)	572 (31)	993 (25)	2010 (19)	3190 (15)	5540 (12)		
7	24 (128)	45 (96)	75 (77)	115 (65)	201 (51)	333 (42)	640 (32)	1110 (25)	2240 (19)	3570 (16)	6200 (13)		
8	27 (131)	49 (99)	83 (79)	127 (67)	222 (53)	368 (43)	706 (33)	1230 (26)	2470 (20)	3940 (16)	6840 (13)		
9	29 (135)	54 (100)	90 (81)	139 (68)	243 (54)	403 (44)	772 (34)	1340 (27)	2710 (20)	4310 (17)	7480 (13)		
10	33 (129)	60 (101)	100 (82)	154 (69)	263 (55)	437 (45)	838 (34)	1450 (27)	2940 (21)	4670 (17)	8110 (14)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c1

Sampling Plans for $\beta = 1/3$, $r = .99$

c	n (t/ρ) x 100 Ratio for which P(A) = .10 (or less) 1500	1000	500	200	100	50	10	5.0	1.0	.50	.10	.050	.010
0	93 (.01)	107 (.01)	134	183	230	288	495	622	1070	1360	2300	2880	4950
1	159 (1.1)	182 (.75)	229 (.37)	309 (.15)	389 (.07)	486 (.04)	837 (.01)	1050	1800	2290	3890	4860	8370
2	217 (5.4)	249 (3.5)	313 (1.8)	422 (.73)	532 (.37)	665 (.18)	1150 (.04)	1440 (.02)	2470	3130	5320	6650	
3	273 (12)	312 (8.4)	393 (4.2)	530 (1.7)	668 (.87)	835 (.44)	1440 (.08)	1810 (.04)	3090 (.01)	3930	[illegible]	8350	
4	326 (22)	374 (15)	470 (7.4)	634 (3.0)	799 (1.5)	999 (.76)	1720 (.15)	2160 (.07)	3700 (.01)	4700	8000	9990	
5	379 (32)	433 (22)	546 (11)	736 (4.5)	928 (2.2)	1160 (1.1)	2000 (.22)	2510 (.11)	4300 (.02)	5460 (.01)	9280		
6	430 (44)	492 (30)	619 (15)	836 (6.1)	1050 (2.8)	1320 (1.5)	2270 (.30)	2850 (.15)	4880 (.03)	6200 (.01)			
7	480 (56)	550 (37)	692 (18)	934 (7.8)	1180 (3.8)	1470 (2.0)	2530 (.38)	3180 (.19)	5450 (.04)	6930 (.02)			
8	530 (68)	607 (46)	764 (23)	1030 (9.4)	1300 (4.7)	1630 (2.4)	2790 (.48)	3510 (.24)	6020 (.05)	7640 (.02)			
9	580 (80)	664 (53)	836 (27)	1130 (11)	1420 (5.6)	1780 (2.8)	3060 (.55)	3840 (.28)	6580 (.06)	8360 (.03)			
10	629 (93)	720 (62)	906 (31)	1220 (13)	1540 (6.4)	1930 (3.2)	3320 (.65)	4170 (.32)	7140 (.06)	9070 (.03)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c2

Sampling Plans for $\beta = 1/2$, $r = .99$

c	n												
	(t/) x 100 Ratio for which P(A) = .10 (or less)												
	1500	1000	500	200	100	50	25	10	5.0	2.5	1.0	.50	.10
0	60 (.71)	73 (.50)	103 (.24)	163 (.10)	230 (.05)	324 (.02)	461 (.01)	683	1030	1540	2330	3250	6840
1	101 (12)	123 (8.1)	175 (4.0)	276 (1.6)	389 (.82)	548 (.41)	778 (.20)	1220 (.08)	1730 (.04)	2430 (.02)	3930 (.01)	5480	
2	138 (35)	170 (23)	240 (11)	377 (4.6)	532 (2.3)	750 (1.2)	1070 (.59)	1660 (.24)	2370 (.12)	3330 (.06)	5380 (.02)	7500 (.01)	
3	175 (60)	213 (40)	301 (20)	474 (8.1)	668 (4.1)	941 (2.0)	1340 (1.0)	2090 (.42)	2970 (.21)	4180 (.10)	6750 (.04)	9410 (.02)	
4	209 (88)	255 (59)	360 (30)	567 (12)	799 (5.9)	1130 (3.0)	1600 (1.5)	2500 (.62)	3550 (.30)	5000 (.15)	8080 (.06)		
5	243 (125)	296 (77)	418 (38)	658 (15)	928 (7.8)	1310 (4.0)	1860 (1.9)	2900 (.80)	4120 (.38)	5800 (.20)	9370 (.08)		
6	276 (140)	336 (94)	474 (47)	747 (19)	1050 (9.6)	1480 (4.8)	2110 (2.4)	3290 (1.0)	4680 (.50)	6580 (.25)			
7	308 (165)	376 (106)	530 (55)	835 (22)	1180 (11)	1660 (5.6)	2360 (2.8)	3680 (1.1)	5230 (.57)	7360 (.28)			
8	340 (190)	415 (125)	585 (65)	921 (26)	1300 (13)	1830 (6.5)	2600 (3.2)	4060 (1.3)	5770 (.65)	8120 (.33)			
9	372 (210)	454 (140)	640 (71)	1010 (28)	1420 (14)	2000 (7.2)	2840 (3.6)	4440 (1.4)	6320 (.73)	8880 (.37)			
10	403 (230)	492 (155)	694 (80)	1090 (31)	1540 (16)	2170 (8.0)	3080 (4.0)	4820 (1.6)	6850 (.80)	9630 (.40)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c3

Sampling Plans for β = 2/3, r = .99

c	n: (t/ρ) x 100 Ratio for which P(A) = .10 or less												
	1500	1000	500	300	200	100	50	25	15	10	5.0	2.5	1.0
0	38 (4.9)	49 (3.3)	79 (1.6)	109 (1.0)	143 (.67)	230 (.33)	366 (.16)	576 (.08)	808 (.05)	1060 (.03)	1690 (.01)	2710	4900
1	64 (41)	84 (30)	133 (14)	185 (8.3)	243 (5.4)	389 (2.8)	617 (1.4)	973 (.69)	1370 (.42)	1790 (.27)	2860 (.14)	4580 (.07)	8200 (.02)
2	87 (91)	114 (60)	181 (30)	253 (18)	333 (12)	532 (6.0)	845 (3.0)	1330 (1.5)	1870 (.90)	2450 (.60)	3910 (.29)	6260 (.15)	
3	110 (137)	144 (92)	227 (46)	318 (27)	418 (18)	668 (9.1)	1060 (4.5)	1670 (2.3)	2340 (1.4)	3080 (.91)	4910 (.45)	7860 (.22)	
4	132 (184)	174 (115)	271 (61)	381 (36)	500 (24)	800 (12)	1270 (6.0)	2000 (3.0)	2800 (1.8)	3680 (1.2)	5880 (.60)	9400 (.30)	
5	155 (215)	202 (145)	314 (75)	442 (45)	580 (30)	928 (15)	1470 (7.4)	2320 (3.7)	3250 (2.3)	4270 (1.5)	6820 (.74)		
6	176 (250)	229 (170)	357 (87)	501 (52)	658 (35)	1050 (17)	1670 (8.6)	2630 (4.4)	3700 (2.6)	4850 (1.7)	7740 (.87)		
7	196 (290)	256 (190)	399 (99)	561 (59)	736 (39)	1180 (19)	1870 (9.7)	2940 (4.9)	4130 (2.9)	5420 (1.9)	8650 (.97)		
8	217 (315)	282 (212)	440 (105)	619 (65)	812 (43)	1300 (21)	2060 (10)	3250 (5.4)	4560 (3.3)	5990 (2.1)	9550 (1.0)		
9	237 (345)	309 (230)	482 (117)	677 (71)	888 (47)	1420 (23)	2260 (10)	3550 (5.8)	4990 (3.4)	6550 (2.3)			
10	257 (370)	335 (250)	522 (126)	734 (76)	963 (50)	1540 (25)	2450 (12)	3850 (6.3)	5410 (3.8)	7100 (2.5)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c4

Sampling Plans for $\beta = 1$, $r = .99$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	1500	1000	800	500	300	200	100	80	50	25	15	10	5.0
0	16 (31)	23 (22)	29 (17)	46 (11)	77 (6.6)	114 (4.5)	230 (2.1)	291 (1.7)	461 (1.1)	903 (.57)	1520 (.34)	2260 (.23)	4610 (.11)
1	27 (132)	40 (88)	49 (72)	78 (45)	130 (27)	194 (18)	389 (9.1)	492 (7.2)	778 (4.6)	1530 (2.3)	2560 (1.4)	3820 (.93)	7780 (.46)
2	37 (225)	54 (153)	68 (120)	107 (76)	179 (45)	266 (30)	532 (15)	674 (12)	1070 (7.7)	2090 (3.9)	3500 (2.3)	5220 (1.6)	
3	46 (305)	68 (203)	85 (162)	135 (100)	225 (60)	334 (40)	668 (20)	846 (16)	1340 (10)	2620 (5.2)	4400 (3.1)	6550 (2.1)	
4	55 (370)	82 (245)	102 (195)	163 (120)	269 (73)	400 (49)	799 (24)	1010 (19)	1600 (12)	3140 (6.3)	5260 (3.7)	7840 (2.5)	
5	64 (422)	95 (280)	118 (225)	189 (138)	312 (83)	464 (56)	928 (28)	1180 (22)	1860 (14)	3640 (7.2)	6100 (4.3)	9100 (2.9)	
6	73 (470)	108 (310)	134 (250)	215 (153)	355 (92)	526 (62)	1060 (31)	1340 (24)	2110 (15)	4130 (7.9)	6930 (4.7)		
7	82 (510)	121 (335)	150 (270)	240 (166)	396 (100)	588 (67)	1180 (33)	1490 (26)	2360 (16)	4620 (8.6)	7750 (5.1)		
8	90 (550)	134 (360)	169 (280)	265 (175)	437 (105)	649 (72)	1300 (36)	1650 (28)	2600 (18)	5100 (9.2)	8550 (5.5)		
9	99 (570)	146 (380)	184 (298)	290 (189)	478 (112)	710 (76)	1420 (38)	1800 (30)	2840 (19)	5570 (9.7)	9350 (5.8)		
10	110 (580)	161 (390)	200 (310)	314 (197)	519 (118)	770 (79)	1540 (39)	1950 (31)	3080 (20)	6050 (10)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c5

Sampling Plans for β = 1 1/3, r = .99

c	n (t/ρ) x 100 Ratio for which P(A) = .10 (or less) 1500	1000	800	500	300	200	150	100	80	50	25	15	10
0	7 (77)	11 (56)	15 (44)	27 (28)	53 (17)	91 (11)	134 (8.5)	230 (5.7)	307 (4.6)	576 (2.8)	1460 (1.4)	2880 (.87)	4900 (.58)
1	11 (245)	19 (162)	26 (127)	46 (81)	90 (49)	155 (32)	227 (24)	389 (16)	519 (13)	973 (8.3)	2460 (4.1)	4860 (2.5)	8280 (1.6)
2	16 (350)	26 (240)	34 (195)	63 (123)	124 (73)	212 (48)	311 (36)	532 (24)	709 (19)	1330 (12)	3370 (6.0)	6650 (3.6)	
3	20 (420)	33 (295)	43 (240)	80 (151)	157 (89)	267 (60)	391 (45)	668 (30)	891 (24)	1670 (15)	4230 (7.3)	8350 (4.4)	
4	24 (510)	39 (345)	52 (275)	95 (175)	187 (104)	320 (68)	467 (52)	799 (34)	1070 (28)	2000 (17)	5060 (8.7)	9990 (5.3)	
5	28 (565)	46 (385)	60 (305)	110 (194)	217 (115)	371 (76)	542 (57)	928 (38)	1240 (31)	2320 (19)	5870 (9.7)		
6	32 (610)	52 (410)	69 (325)	126 (210)	247 (124)	421 (82)	616 (61)	1050 (41)	1410 (33)	2630 (21)	6670 (10)		
7	36 (650)	58 (440)	77 (350)	141 (220)	276 (132)	471 (87)	688 (65)	1180 (43)	1570 (35)	2940 (22)	7450 (11)		
8	40 (680)	65 (458)	85 (367)	157 (225)	305 (139)	520 (92)	760 (69)	1300 (46)	1730 (37)	3250 (23)	8220 (11)		
9	43 (720)	70 (485)	93 (383)	172 (235)	333 (144)	568 (95)	831 (72)	1420 (48)	1900 (38)	3550 (24)	9000 (12)		
10	47 (730)	78 (490)	103 (385)	187 (245)	362 (150)	616 (100)	[illegible]01 ([illegible]3)	1540 (50)	2060 (40)	3850 (25)	9750 (12)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or less)

Table 3c6

Sampling Plans for β = 1 2/3, r = .99

c	n												
	$(t/\rho) \times 100$ Ratio for which P(A) = .10 (or less)												
	1000	800	500	300	250	200	150	100	80	50	40	25	15
0	5 (101)	8 (76)	16 (50)	37 (30)	50 (25)	73 (20)	117 (15)	230 (10)	334 (8.2)	731 (5.1)	1050 (4.1)	2300 (2.6)	5360 (1.5)
1	9 (237)	13 (187)	27 (117)	63 (70)	85 (59)	123 (47)	198 (35)	389 (23)	564 (19)	1240 (14)	1770 (9.5)	3890 (6.0)	9050 (3.7)
2	13 (316)	18 (255)	38 (160)	86 (95)	116 (81)	169 (64)	272 (48)	532 (32)	771 (26)	1690 (16)	2420 (13)	5320 (8.0)	
3	16 (385)	23 (303)	47 (193)	108 (115)	146 (95)	213 (76)	341 (57)	668 (38)	968 (30)	2120 (19)	3040 (15)	6680 (9.5)	
4	20 (420)	27 (345)	57 (215)	130 (129)	176 (106)	254 (85)	408 (64)	799 (43)	1158 (34)	2540 (21)	3630 (17)	8000 (11)	
5	23 (459)	32 (370)	66 (236)	150 (141)	205 (115)	295 (92)	473 (69)	928 (46)	1350 (37)	2950 (23)	4220 (19)	9280 (11)	
6	26 (490)	36 (395)	75 (249)	172 (148)	233 (123)	335 (98)	537 (74)	1060 (49)	1530 (39)	3340 (24)	4790 (20)		
7	29 (510)	41 (410)	84 (262)	193 (154)	260 (129)	375 (104)	600 (77)	1180 (51)	1710 (41)	3740 (26)	5350 (21)		
8	32 (535)	45 (430)	92 (272)	213 (160)	287 (134)	414 (107)	663 (81)	1300 (54)	1880 (43)	4130 (27)	5910 (22)		
9	36 (540)	49 (445)	101 (261)	233 (166)	314 (139)	452 (111)	725 (83)	1420 (56)	2060 (45)	4510 (28)	6460 (22)		
10	40 (540)	54 (445)	112 (283)	253 (171)	341 (143)	491 (115)	786 (86)	1540 (57)	2230 (46)	4890 (28)	7010 (23)		

$(t/\rho) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

Table 3c7

Sampling Plans for β = 2, r = .99

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	800	500	400	300	250	200	150	120	100	80	50	40	25
0	4 (110)	10 (71)	15 (58)	26 (43)	37 (37)	58 (30)	102 (22)	156 (17)	230 (15)	349 (12)	903 (7.4)	1,400 (5.9)	3,600 (3.7)
1	7 (230)	16 (150)	25 (120)	44 (90)	63 (74)	98 (60)	169 (45)	263 (36)	389 (29)	589 (24)	1530 (15)	2,360 (12)	6,080 (7.5)
2	10 (300)	23 (190)	34 (150)	60 (110)	86 (97)	134 (77)	232 (58)	360 (47)	532 (39)	806 (31)	2,090 (19)	3,230 (16)	8,320 (10)
3	13 (340)	29 (220)	43 (180)	76 (130)	108 (110)	167 (90)	291 (67)	452 (54)	668 (45)	1,010 (36)	2,620 (22)	4,050 (18)	
4	15 (380)	34 (240)	52 (190)	91 (140)	130 (120)	200 (99)	348 (74)	540 (60)	799 (49)	1,210 (40)	3,140 (25)	4,850 (21)	
5	18 (410)	40 (260)	60 (210)	105 (160)	150 (130)	232 (100)	403 (79)	627 (64)	928 (52)	1,410 (43)	3,640 (26)	5,620 (21)	
6	20 (440)	45 (270)	69 (220)	120 (160)	170 (140)	263 (110)	458 (83)	712 (67)	1,050 (54)	1,600 (45)	4,130 (28)	6,380 (22)	
7	23 (450)	51 (280)	77 (230)	134 (170)	190 (140)	294 (110)	512 (87)	795 (70)	1,180 (58)	1,780 (47)	4,620 (29)	7,130 (23)	
8	25 (470)	57 (290)	85 (240)	148 (180)	210 (150)	325 (120)	565 (90)	878 (73)	1,300 (59)	1,970 (48)	5,090 (30)	7,870 (24)	
9	28 (475)	62 (300)	93 (240)	162 (182)	229 (153)	355 (122)	618 (92)	960 (74)	1420 (61)	2150 (50)	5570 (31)	8610 (25)	
10	32 (480)	69 (305)	103 (245)	175 (187)	249 (155)	385 (125)	670 (94)	1040 (76)	1540 (62)	2340 (51)	6040 (32)	9340 (25)	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c8

Sampling Plans for β = 2 1/2, r = .99

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	500	400	300	250	200	150	120	100	80	65	50	40	25
0	5 (101)	8 (83)	15 (64)	24 (53)	41 (43)	84 (32)	146 (26)	230 (22)	397 (17)	668 (14)	1320 (11)	2300 (8.7)	7430 (5.4)
1	8 (185)	13 (150)	26 (114)	40 (95)	69 (76)	141 (57)	248 (46)	389 (38)	671 (31)	1130 (25)	2230 (19)	3890 (15)	
2	11 (230)	18 (185)	35 (141)	55 (118)	95 (93)	195 (70)	339 (56)	532 (47)	918 (38)	1540 (31)	3040 (23)	5320 (18)	
3	14 (260)	23 (208)	45 (156)	69 (132)	119 (105)	245 (78)	426 (63)	668 (53)	1150 (42)	1940 (34)	3820 (26)	6680 (21)	
4	17 (280)	27 (225)	53 (170)	83 (142)	143 (114)	293 (84)	509 (68)	799 (56)	1380 (46)	2320 (37)	4570 (28)	8000 (22)	
5	20 (295)	32 (238)	62 (178)	96 (150)	168 (119)	340 (89)	591 (71)	928 (60)	1600 (48)	2690 (39)	5300 (30)	9280 (24)	
6	22 (312)	36 (250)	71 (185)	110 (155)	190 (124)	386 (93)	671 (74)	1050 (62)	1820 (50)	3050 (41)	6020 (31)		
7	25 (320)	41 (255)	79 (193)	123 (160)	213 (127)	431 (96)	750 (77)	1180 (64)	2030 (52)	3410 (42)	6730 (32)		
8	28 (328)	45 (265)	87 (199)	135 (165)	235 (132)	476 (98)	827 (79)	1300 (66)	2240 (53)	3770 (43)	7420 (33)		
9	31 (334)	49 (271)	95 (203)	148 (168)	257 (135)	521 (101)	905 (80)	1420 (67)	2450 (54)	4120 (44)	8120 (34)		
10	34 (335)	54 (275)	105 (204)	163 (170)	279 (137)	564 (103)	982 (82)	1540 (69)	2660 (55)	4470 (45)	8810 (34)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 4-a

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .50

	c	Sample Size - n						
		10	25	50	100	250	500	1000
	0	.18	.030	.0076	.0019	.00030	.000076	.000019
	1	.51	.080	.019	.0047	.00080	.00019	.000047
$\beta=\frac{1}{2}$	2	1.0	.14	.034	.0084	.0014	.00034	.000083
	3	1.7	.23	.054	.013	.0021	.00050	.00012
	4	2.7	.33	.077	.018	.0029	.00071	.00017
	5	4.4	.46	.097	.024	.0039	.00093	.00023
	0	.32	.095	.038	.015	.0046	.0018	.00071
	1	.66	.18	.070	.028	.0085	.0033	.0013
$\beta=\frac{3}{4}$	2	1.0	.27	.10	.041	.012	.0049	.0019
	3	1.4	.37	.14	.054	.016	.0065	.0025
	4	2.0	.48	.18	.068	.020	.0081	.0031
	5	2.8	.60	.21	.083	.025	.0097	.0038
	0	.42	.17	.085	.043	.017	.0087	.0043
	1	.72	.27	.13	.068	.028	.014	.0069
$\beta=1$	2	1.0	.38	.18	.090	.036	.018	.0090
(Exp.)	3	1.3	.47	.23	.11	.045	.022	.011
	4	1.7	.57	.27	.13	.053	.026	.013
	5	2.1	.67	.31	.15	.061	.030	.015
	0	.53	.27	.15	.095	.048	.028	.017
	1	.78	.38	.22	.13	.067	.040	.023
$\beta=1\frac{1}{3}$	2	1.0	.48	.28	.16	.083	.049	.029
	3	1.2	.57	.33	.19	.098	.058	.034
	4	1.4	.66	.38	.22	.11	.065	.038
	5	1.7	.75	.41	.24	.12	.070	.043
	0	.59	.35	.23	.15	.088	.057	.038
	1	.81	.46	.30	.20	.11	.075	.050
$\beta=1\frac{2}{3}$	2	1.0	.55	.36	.24	.13	.090	.059
	3	1.2	.64	.41	.27	.15	.10	.067
	4	1.3	.71	.45	.30	.17	.11	.074
	5	1.5	.78	.49	.32	.18	.12	.080
	0	.65	.41	.29	.21	.13	.093	.066
	1	.85	.52	.36	.26	.16	.11	.082
$\beta=2$	2	1.0	.61	.42	.30	.19	.13	.095
	3	1.1	.69	.47	.33	.21	.15	.10
	4	1.3	.76	.52	.36	.23	.16	.11
	5	1.4	.82	.55	.39	.24	.17	.12
	0	.70	.50	.37	.28	.19	.15	.11
	1	.88	.59	.45	.34	.23	.18	.13
$\beta=2\frac{1}{2}$	2	1.0	.67	.50	.38	.26	.20	.15
	3	1.1	.73	.55	.41	.28	.22	.16
	4	1.2	.79	.59	.44	.30	.23	.17
	5	1.3	.85	.62	.47	.32	.24	.18

Table 4-b

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .90

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	8.2	1.3	.32	.081	.013	.0032	.00080
	1	22	3.3	.82	.20	.033	.0080	.0020
	2	43	6.2	1.5	.36	.059	.014	.0035
	3	72	10	2.3	.55	.090	.022	.0053
	4	110	15	3.3	.78	.12	.031	.0074
	5	180	20	4.1	1.0	.16	.040	.0099
$\beta=\frac{3}{4}$	0	4.0	1.2	.46	.18	.056	.022	.0087
	1	8.0	2.2	.87	.34	.10	.040	.016
	2	12	3.4	1.3	.50	.15	.060	.023
	3	18	4.6	1.7	.67	.20	.079	.031
	4	24	5.9	2.2	.85	.25	.099	.038
	5	34	7.3	2.6	1.0	.30	.12	.047
$\beta=1$ (Exp.)	0	2.8	1.1	.57	.28	.11	.057	.028
	1	4.7	1.8	.90	.45	.18	.091	.045
	2	6.5	2.5	1.2	.60	.24	.12	.060
	3	8.7	3.1	1.5	.75	.30	.15	.074
	4	11	3.7	1.8	.88	.35	.17	.088
	5	14	4.4	2.0	1.0	.41	.20	.10
$\beta=1\frac{1}{3}$	0	2.2	1.1	.65	.38	.20	.11	.068
	1	3.2	1.5	.92	.55	.28	.16	.097
	2	4.1	2.0	1.2	.67	.34	.20	.12
	3	5.1	2.3	1.3	.80	.40	.24	.14
	4	6.2	2.7	1.5	.91	.45	.27	.16
	5	7.2	3.1	1.7	1.0	.51	.30	.18
$\beta=1\frac{2}{3}$	0	1.8	1.0	.71	.47	.27	.18	.12
	1	2.5	1.4	.93	.62	.36	.24	.15
	2	3.1	1.7	1.1	.73	.43	.28	.18
	3	3.7	2.0	1.2	.83	.48	.32	.21
	4	4.3	2.2	1.4	.92	.53	.35	.23
	5	4.9	2.4	1.5	1.0	.58	.38	.25
$\beta=2$	0	1.6	1.0	.76	.53	.34	.24	.17
	1	2.1	1.3	.95	.67	.43	.30	.21
	2	2.6	1.5	1.1	.78	.50	.35	.24
	3	3.0	1.7	1.2	.85	.55	.38	.27
	4	3.3	1.9	1.3	.95	.60	.42	.29
	5	3.8	2.1	1.4	1.0	.64	.45	.32
$\beta=2\frac{1}{2}$	0	1.5	1.0	.79	.60	.42	.32	.24
	1	1.8	1.2	.96	.72	.50	.38	.29
	2	2.1	1.4	1.1	.81	.57	.43	.32
	3	2.4	1.5	1.2	.88	.62	.47	.35
	4	2.6	1.7	1.2	.96	.66	.50	.37
	5	2.9	1.8	1.3	1.0	.70	.53	.39

Table 4-c

Minimum Lifetesting Times to Assure Lot Compliance
with 95% Confidence

In Multiples of Specified Reliable Life -- r = .99

		Sample Size - n						
	c	10	25	50	100	250	500	1000
	0	850	140	35	9.0	1.4	.35	.090
	1	2400	360	90	23	3.6	.90	.22
$\beta=\frac{1}{2}$	2	4700	670	160	39	6.3	1.5	.39
	3	8500	1000	250	60	9.8	2.4	.60
	4	13000	1500	360	86	13	3.4	.83
	5	22000	2100	450	110	19	4.4	1.1
	0	91	27	10	4.2	1.3	.50	.20
	1	180	52	20	7.9	2.3	.94	.37
$\beta=\frac{3}{4}$	2	290	77	30	11	3.4	1.3	.53
	3	410	100	40	15	4.6	1.8	.70
	4	560	130	50	19	5.7	2.2	.87
	5	770	170	59	23	6.9	2.7	1.0
	0	29	12	5.9	3.0	1.2	.59	.29
	1	51	19	9.5	4.8	1.9	.95	.47
$\beta=1$	2	70	26	12	6.3	2.5	1.2	.63
(Exp.)	3	92	33	16	7.8	3.1	1.5	.77
	4	110	39	19	9.3	3.6	1.8	.90
	5	150	47	21	10	4.3	2.1	1.0
	0	12	6.3	3.7	2.2	1.1	.67	.40
	1	18	9.1	5.4	3.2	1.6	.96	.56
$\beta=1\frac{1}{3}$	2	24	11	6.7	3.9	2.0	1.2	.70
	3	29	13	7.8	4.6	2.3	1.4	.81
	4	34	15	9.0	5.3	2.6	1.6	.92
	5	41	17	9.8	5.9	2.9	1.7	1.0
	0	7.5	4.4	2.9	1.9	1.1	.73	.48
	1	10	5.8	3.8	2.5	1.5	.96	.64
$\beta=1\frac{2}{3}$	2	12	7.0	4.6	3.0	1.7	1.1	.75
	3	15	7.9	5.2	3.4	2.0	1.3	.85
	4	17	9.0	5.7	3.8	2.2	1.4	.94
	5	20	10	6.2	4.1	2.4	1.5	1.0
	0	5.5	3.4	2.4	1.7	1.1	.77	.54
	1	7.1	4.3	3.0	2.1	1.3	.96	.67
$\beta=2$	2	8.4	5.1	3.6	2.5	1.6	1.1	.78
	3	9.6	5.8	4.0	2.8	1.7	1.2	.87
	4	11	6.3	4.3	3.0	1.9	1.3	.95
	5	12	6.9	4.6	3.2	2.0	1.4	1.0
	0	3.8	2.7	2.0	1.5	1.0	.80	.61
	1	4.7	3.2	2.4	1.8	1.3	.97	.73
$\beta=2\frac{1}{2}$	2	5.4	3.6	2.7	2.0	1.4	1.1	.83
	3	6.0	4.0	3.0	2.2	1.5	1.2	.89
	4	6.7	4.3	3.2	2.4	1.6	1.2	.95
	5	7.3	4.6	3.4	2.6	1.7	1.3	1.0

Table 5

Minimum Lifetesting Times to Assure Lot Compliance with 95% Confidence

In Multiples of Specified Minimum Mean Life

	c	Sample Size - n						
		10	25	50	100	250	500	1000
$\beta=\frac{1}{2}$	0	.045	.0071	.0018	.00045	.000076	.000018	.000004
	1	.12	.018	.0045	.0011	.00018	.000047	.000011
	2	.25	.034	.0080	.0020	.00033	.000084	.000020
	3	.44	.055	.012	.0031	.00050	.00012	.000030
	4	.73	.080	.018	.0043	.00069	.00017	.000041
	5	1.20	.11	.023	.0058	.00095	.00022	.000057
$\beta=\frac{3}{4}$	0	.17	.049	.019	.0078	.0023	.00091	.00037
	1	.34	.092	.037	.014	.0042	.0017	.00067
	2	.53	.14	.054	.021	.0063	.0025	.00098
	3	.77	.19	.073	.027	.0083	.0032	.0013
	4	1.00	.25	.091	.035	.010	.0040	.0016
	5	1.30	.31	.11	.041	.012	.0049	.0019
$\beta=1$ (Exp.)	0	.29	.12	.060	.030	.012	.0060	.0030
	1	.50	.19	.095	.048	.019	.0097	.0048
	2	.70	.26	.13	.063	.025	.013	.0064
	3	.94	.33	.16	.078	.031	.016	.0077
	4	1.20	.40	.19	.093	.037	.018	.0091
	5	1.50	.47	.21	.11	.043	.021	.010
$\beta=1\frac{1}{3}$	0	.44	.22	.13	.077	.040	.023	.014
	1	.65	.32	.18	.11	.056	.033	.019
	2	.83	.40	.23	.13	.069	.041	.024
	3	1.00	.48	.27	.16	.080	.048	.028
	4	1.20	.55	.31	.18	.091	.054	.032
	5	1.50	.62	.34	.20	.10	.060	.036
$\beta=1\frac{2}{3}$	0	.54	.31	.20	.13	.079	.052	.034
	1	.75	.41	.27	.18	.10	.068	.045
	2	.90	.50	.32	.21	.12	.082	.053
	3	1.00	.58	.37	.24	.14	.093	.060
	4	1.20	.65	.41	.27	.15	.10	.067
	5	1.50	.71	.44	.29	.17	.11	.073
$\beta=2$	0	.62	.39	.27	.19	.12	.087	.062
	1	.80	.49	.35	.24	.15	.11	.078
	2	.95	.58	.40	.28	.18	.13	.089
	3	1.00	.65	.45	.31	.20	.14	.10
	4	1.20	.72	.49	.34	.21	.15	.11
	5	1.40	.78	.52	.37	.23	.16	.12
$\beta=2\frac{1}{2}$	0	.69	.48	.36	.27	.19	.15	.11
	1	.85	.58	.43	.33	.23	.17	.13
	2	.98	.66	.49	.37	.26	.19	.15
	3	1.10	.72	.53	.40	.28	.21	.16
	4	1.20	.77	.57	.43	.30	.23	.17
	5	1.30	.82	.60	.46	.32	.24	.18

Table 6

Values of $L_{.95}/L_{.05} = \mu_{.95}/\mu_{.05}$

c	$\beta = 1/2$	$\beta = 3/4$	$\beta = 1$	$\beta = 1\ 1/3$	$\beta = 1\ 2/3$	$\beta = 2$	$\beta = 2\ 1/2$
0	3,300	220	58	21	11	7.6	5.0
1	170	31	13	7.0	4.7	3.6	2.8
2	59	15	7.7	4.6	3.4	2.8	2.3
3	32	10	5.7	3.6	2.8	2.4	2.0
4	22	7.6	4.6	3.1	2.5	2.2	1.8
5	16	6.2	4.0	2.8	2.3	2.0	1.7

$L_{.95}$ = reliable life, $\mu_{.95}$ = mean life for which $P(A) = .95$

$L_{.05}$ = reliable life, $\mu_{.05}$ = mean life for which $P(A) = .05$

Appendix A

Reliable Life as a Life-quality Criterion

This appendix describes the concept of reliable life or quantile life of complementary order which is used as the life-quality criterion for items subject to the testing procedures given in this report.

For an arbitrary lifelength distribution defined over $\gamma \leq x < \infty$ (γ is the threshold or location parameter) with c.d.f. = $F(x)$ and p.d.f. = $f(x)$, the reliability function = $R(x)$ = $1-F(x)$ and a reliability index r $(0 < r < 1)$, the reliable life ρ_r (see Reference 12) is the solution of x in $R(x) = r$ or,

$$\rho_r = R^{-1}(r) \tag{A1}$$

where R^{-1} is the inverse function of R.

If the lifelength of an item follows a Weibull distribution of the form:

$$\begin{aligned} F(x) &= 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], \quad x \geq \gamma;\ \eta,\ \beta > 0\ ; \\ &= 0, \text{ otherwise} \end{aligned} \tag{A2}$$

and its p.d.f.,

$$\begin{aligned} f(x) &= \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], \quad x \geq \gamma;\ \eta,\ \beta > 0\ ; \\ &= 0, \text{ otherwise,} \end{aligned} \tag{A3}$$

then the reliable life or order r will be,

$$\rho_r = \gamma + \eta\,(-\ln r)^{b} \tag{A4}$$

where $b = 1/\beta$. In this report, since γ is assumed to be known, Equations (A2, A3 & A4) are not used. Instead, Equations (A5, A6, & A7) are used.

For γ known, there is no loss of generality by assuming $\gamma = 0$. In this case,

$$F(x) = 1 - \exp[-(x/\eta)^{\beta}] \quad , \tag{A5}$$

$$f(x) = \frac{\beta}{\eta}(x/\eta)^{\beta-1} \exp[-(x/\eta)^{\beta}] \quad , \tag{A6}$$

and the reliable life is

$$\rho_r = \eta(-\ln r)^b \quad . \tag{A7}$$

Now let the testing time be truncated at t and let p' be the probability of failure of an item prior to t, then combining (A5) and (A7),

$$p' = F(t) = 1 - \exp\{-[t(-\ln r)^b/\rho_r]^{\beta}\} \quad , \tag{A8}$$

which can be simplified as,

$$p' = 1 - \exp[(t/\rho_r)^{\beta} \ln(r)] \quad . \tag{A9}$$

It can be noted now that if the truncation time coincides with ρ_r, one would always (for any $\beta > 0$) have $p' = 1-\exp[\ln(r)] = 1 - r$, which is to be expected. Also since $0 < r < 1$, $\ln(r)$ will always be negative and finite; thus the Weibull c.d.f. in the form of Equation (A8) or (A9) satisfies the conditions: $F(0) = 0$ and $F(\infty) = 1$ and the c.d.f. is monotonic in t for $\beta > 0$.

The inverse of Equation (A9) gives,

$$t/\rho_r = [\ln(1-p') / \ln(r)]^b \tag{A10}$$

Notice that Equation (A10) also asserts that for any $\beta > 0$, $p' = 1-r$ if $t = \rho_r$.

References

1. Quality Control and Reliability Technical Report TR3, Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Mean Life Criterion), Office of the Assistant Secretary of Defense (Installations and Logistics), U.S. Government Printing Office, 1961.

2. Quality Control and Reliability Technical Report TR4, Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Hazard Rate Criterion), Office of the Assistant Secretary of Defense (Installations and Logistics), U.S. Government Printing Office, 1962.

3. Goode, Henry P. and Kao, John H.K., "Sampling Plans Based on the Weibull Distribution," Proceedings of the Seventh National Symposium on Reliability and Quality Control, 1961, pp. 24-40.

4. Goode, Henry P. and Kao, John H.K., "Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Hazard Rate Criterion)," Proceedings of the Eighth National Symposium on Reliability and Quality Control, 1962, pp. 37-58.

5. Goode, Henry P. and Kao, John H.K., "An Adaptation of the MIL-STD-105B Plans to Reliability and Life Testing Applications," Transactions of the Fifteenth Annual Convention of the American Society for Quality Control, 1961, pp. 245-259.

6. Goode, Henry P. and Kao, John H.K., "Use of the Military Standard Plans for Hazard Rate Under the Weibull Distribution," Transactions of the Sixteenth Annual Convention of the American Society for Quality Control, 1962, pp. 371-389.

7. Kao, John H.K., "A Summary of Some New Techniques on Failure Analysis," Proceedings of the Sixth National Symposium on Reliability and Quality Control in Electronics, 1960, pp. 190-201.

8. Plait, Alan, "The Weibull Distribution -- with Tables," Industrial Quality Control, Vol. XIX, November, 1962, pp. 17-26.

9. Cramer, Harald, "Mathematical Methods of Statistics," Princeton University Press, Princeton, N.J., 1946.

10. Finney, D.J., Statistical Method in Biological Assaying, Charles Griffin (London), 1952.

11. ASTM STP-91A, A Tentative Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data, 1958. (A revised version including techniques using the Weibull distribution will be published soon.)

12. Weibull, W., Fatigue Testing and the Analysis of Results, Pergamon Press, 1961.

13. Lieblein, J. and Zelen, M., "Statistical Investigation of the Fatigue Life of Deep-groove Ball Bearings," Journal of Research, National Bureau of Standards, Vol. 57, No. 5, 1956, pp. 273-316.

14. Moyer, C.A., et al., "The Weibull Distribution Function for Fatigue Life," Materials Research and Standards, Vol. 2, No. 5, May, 1962.

15. Cameron, J.M., Tables for Constructing and for Computing the Operating Characteristics of Single-Sampling Plans," Industrial Quality Control, July, 1952, pp. 37-39.

16. Kao, John H.K., "The Beta Distribution in Reliability and Quality Control," Proceedings of the Seventh National Symposium on Reliability and Quality Control, 1961, pp. 496-511.

17. Grubbs, Frank E., "On Designing Single Sampling Inspection Plans," The Annals of Mathematical Statistics, Vol. 20, 1949, pp. 242-256.

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